ESSAYS ON OPTIMAL INVESTMENTS IN THE MITIGATION OF GLOBAL WARMING

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Chapter 1

Introduction

The potential damages of Climate Change are considered today as almost certain if concrete steps against it are not financed and put in place by governments around the world. Raising sea levels, droughts, unpredictable weather events, usually with a potential never recorded in the past, are few among many consequences of raising temperatures and changing climate. In light of this, politicians, non-governmental organizations and a growing share of the civil society are engaging to make binding greenhouse gas emission reductions part of international agreements. After years of scarce results, at the end of 2015, the 21st Conference of the Parties held in Paris saw developed and developing nations agree on reducing greenhouse gas (GHG) emissions and limiting the global average temperature increase to 1.5°C. This represents a very important result, as increases in global average temperatures have important negative consequences on the environment, which ultimately affect the financial health of a country. In fact, possible temperature increases and consequent catastrophes can have serious effects on a country's and on the world's financial stability. It is then important to estimate the financial impacts of such catastrophes.

In the first part of this manuscript, we model the negative externalities of a positive-drifted stochastic temperature process on the global Gross Domestic Product. We analyse the choices of a decision maker in terms of when to invest a fraction of the global GDP and at what temperature level, when confronted with a potential environmental catastrophe. In a real options setting, we find that the optimal temperature level at which to invest a fraction of GDP is dependent on the uncertainty surrounding the temperature process. The investment threshold is an increasing function of the volatility when the latter is small. Conversely, it is a decreasing function of the volatility for higher levels of the latter, due to a greater risk of a potential climate catastrophe. For what concerns the fraction of GDP to be invested in mitigation, it increases with volatility, meaning that more uncertainty causes more expenditures in mitigation as the temperature process has to be controlled more powerfully, but it is always positive and at the optimum higher than adaptation expenditures, proving that the decision maker cannot chose not to intervene.

Having found and quantified the links between raising temperatures, GDP and mitigation, we move our attention to the choices that a decision maker has in terms of emission reduction strategies. However, the set of choices is vast, and these are extremely heterogeneous in terms of funding needed and GHG reduction potential. Moreover, it is important that mitigation strategies are designed around each country's characteristics. In fact, each country, in particular in the developing world, has its own peculiarities, and unique/global strategies can prove unsuccessful or damaging if not designed around each country's needs, goals and reduction potential. Mitigation strategies then differ according to the country or area of analysis. As an example, among the biggest contributors to global greenhouse gas concentrations we find deforestation and energy consumption, the first affecting mainly South America and some areas of Asia, such as Indonesia and Malaysia, the second particularly evident in the economies in transition in South East Asia, such as India or China, or in the most developed countries around the world. As a consequence, avoiding deforestation and forest degradation and improving energy efficiency have been considered by experts as two very powerful ways of mitigating climate change, given their very high GHG reduction potential. This thesis wants to contribute to both causes. Indeed, in the second part, we analyse the optimal choices of a forester that has the chance to enter a Reducing Emissions from Deforestation and Forest Degradation (REDD) scheme, thus protecting its forest and receiving, as compensation for avoiding deforestation, a stochastic REDD permit price. We analyse how the decision to participate in the scheme changes when different values of the choice variables are taken.

We find that when the forester is allowed to deforest at his own optimal rate, he cuts his entire forest in just a few years. Conversely, when deforestation rates are bounded from above, he finds optimal to enter a REDD scheme in finite time. These results are confirmed even when risk aversion is taken into consideration. The more risk averse is the forester, the later he optimally enters a REDD scheme, thus deforesting more land. Such results confirm the usefulness of an intermediary agent in the market, such as a REDD manager, that bears the risk of a stochastic REDD price. Only in this case it is possible to incentivize the forester to participate in the scheme sooner, thus saving a bigger portion of its forest.

Finally, in the third part, we analyse a financing scheme whose goal is subsidizing the production of super energy efficient fans. Tackling climate change and promoting investments that award energy savings and efficiency is particularly important in the developing world and in South East Asia in the specific. Here coal power plants have to satisfy the need for electricity of a growing number of citizens. In China and India, a green revolution could mean a huge amount of polluting emissions reduced at low prices, with a great value for the buck. A growing number of international institutions, NGOs and development banks is channelling investments for the development of energy savings projects, mainly in the form of improved efficiency. One example is the Super Efficient Equipment Program in India, which is financially sponsored by the Clean Technology Fund with an amount of \$50,000,000 and whose goal is a reduction in electricity consumption of 19,000 Megawatts in three years, by introducing a subsidy for the production of super efficient electric fans in India. We see that the barriers that such financing scheme has to face are too strong for a subsidy to work. Our results show that, due to a particular structure of the Indian retail markets, an important share of funds is wasted, and the reductions achieved in terms of electricity, and consequently greenhouse gas emissions, are negligible.

Chapter 2

Market Uncertainty and Risk Transfer in REDD Projects

Joint work with Marc Chesney and Jonathan Gheyssens

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Abstract

The central role played by deforestation in the increase in global CO_2 emissions has recently justified the development of new schemes which offer compensation in exchange for reductions in emissions from deforestation (Reducing Emissions from Deforestation and Forest Degradation, REDD). The design of REDD projects can be based on market prices to set compensation terms. With limited experiments involving a true market integration of REDD, it remains however difficult to assess the potential impact market price uncertainties may have on the targets of the protective scheme.

The goal of this article is to assess the optimal choices of a forester, in terms of deforestation rate and time length, given his option to enter an irreversible REDD scheme that provides him with stochastic cash flows under different risk aversion scenarios.

Keywords: *Climate Change, Deforestation, REDD, Real Options, Risk Aversion*

2.1 Introduction

According to the IPCC (IPCC, 2014), land use change and forestry activities are among the largest conveyors of greenhouse gases (GHG) and are likely responsible for 15% to 20% of global emissions in recent years.

Triggered by an increase in commodity prices, lack of land tenure and local consumption, world forest cover experienced an average loss of 7.6 million hectares per annum from 2010 to 2015 (FAO, 2015), mostly in tropical areas that support large biodiversity hotspots (albeit with current reduction in the observed deforestation for Brazil and Indonesia).

The removal of forests has various well-documented, negative environmental impacts including: damages to habitat and biodiversity loss, aridity, soil erosion and ecosystem disruption. These impacts are compounded by the fact that tropical forests are recognized as highly efficient carbon sink, capable of holding 50% more carbon per hectare than forests in temperate and boreal areas (IPCC, 2014). This advantage has prompted efforts to integrate avoided deforestation in finance mechanisms against climate change, albeit with mixed results.

Under the Kyoto Protocol, the clean development mechanism (CDM) allows for afforestation and reforestation projects and so far has been initially preferred to the development of projects targeting the reduction in emission from deforestation and degradation (REDD). The reason for this can be found in technical and methodological issues, such as measurability, baseline calculation and risk of leakage.

A REDD scheme awards those foresters whose deforestation rate is lower than a given baseline, with tradable permits. Such permits can be marketed and sold to the foresters who did not manage to keep their deforestation rate below a baseline. By doing so, the seller of the REDD permits is compensated for *avoiding deforestation.* Moreover, REDD schemes, which are expected to provide a large volume of permits at lower marginal price, could prove especially interesting to the companies that find reducing emissions internally to be prohibitively expensive or those needing to buy an important volume of permits (Peters-Stanley et al., 2013).

The Paris Conference of Parties (COP-21) has restated the importance of building capacity for REDD projects with the adoption of new Sustainable Development Goals (SDGs). Indeed, through the SDGs, the Conference of the Parties *explicitly recognizes the importance of forests, urging the international community to "protect, restore and promote sustainable use of terrestrial ecosystems, sustainably manage forests, combat desertification, and halt and reverse land* *degradation and biodiversity loss". REDD actions are already contributing to the achievement of these objectives.*

If projects are to be developed according to the commonly accepted blueprints, one of the most crucial changes from previous attempts to compensate reduced deforestation will be the involvement of carbon markets and their volatile prices. While price uncertainty is not new to forester managers (see section 2.2), the effects of stochastic payments for environmental services (PES) on REDD scheme performance is still very much in debate.

Our aim in this paper is to analyse and to model the optimal timing of a REDD investment. As detailed in section 2.4, our model involves the combined decision of choosing when to join the scheme (knowing that the decision to protect is irreversible), and the deforestation rate prior to it. The trade-off being that higher prior revenues from extraction limit the size of forest to protect and hence the potential PES revenues from REDD.

Our model accounts for the likely presence of risk aversion amongst foresters, a characteristic that could change the attractiveness of the REDD scheme, i.e. by reducing the willingness to enter into it, while improving the risk transfer potential of a forester-manager configuration. In fact, as Parks (1995) has already highlighted, risk aversion is one of the main drivers of land use changes and deforestation. Understanding how risk aversion influences the optimal decision of a forester could also improve also the risk transfer potential of a forester-manager configuration. Furthermore, our model justifies the existence of a REDD manager (along the lines of the protected area management (PAM) model) who can serve as a middle-man between the REDD market and the forester. Therefore, in the context of our dynamic, stochastic model, we test the influence of the REDD managers whose presence is often justified by their ability to shield the foresters from price volatility (by proposing instead a deterministic price formula), thus providing an effective instrument to overcome those issues arising from the forester's risk aversion profile and which can hinder the success of a REDD scheme.

In order to solve our dynamic model with and without the presence of risk aversion, we rely on a real options setting, as described in general terms by Pindyck (2000) and in the case of forestry investments by Insley (2002) and Kassar and Lasserre (2004). This choice is well-suited in case of uncertainty about future cash flows and future risks (Baranzini et al., 2003). Moreover, since the decision to enter a forest protection scheme has often been treated as a once-and-for-all decision by the literature, real options allows us to identify the optimal entry time in such investment, while taking into account the irreversibility of the commitment (Chesney et al., 2016).

Real option models are particularly well adapted to such optimal stopping problems. They are used in order to check whether or not investment decisions should be taken. The standard tool used in this setting before real options were introduced is the Net Present Value (NPV in what follows) approach according to which, an investment should be realized if and only if its NPV, i.e. the difference between its expected discounted payoffs and costs, is positive. This latter criteria is static to the extent to which the choice is between realizing the investment at the date when the NPV is calculated, or never. This is a significant drawback of the NPV criterion. If investment opportunities are considered as real options, the investor has the right, and not the obligation, to make an investment during a given period of time. When identifying the optimal investment date, the possibility of postponing the investment is taken into account. See Dixit and Pindyck (1994) for an excellent reference. Usually, the timing of the investment is the objective of these models. In fact the concept of option value was introduced in environmental economics before the appearance of real options (Arrow and Fisher, 1974; Henry, 1974; Fisher and Krutilla, 1975). It stresses that performing an irreversible action at one point in time involves the cost of renouncing the flexibility to wait; if this cost is correctly taken into account in a cost benefit analysis, in order for the action to be economically justified, the benefits from the decision must be higher than in a traditional cost benefit analysis.

Results show that the forester has no incentive to enter into a REDD scheme in the case where he can freely choose how much of his land to deforest. However, when the authority sets limits on the annual deforestation rate, he participates in the REDD scheme in a finite time and with a positive fraction of forest left for protection. Results are also dependent on the discount rate. Lower discount rates postpone the decision to enter into a REDD scheme, while higher ones, as is the case in many developing countries, prove to be an incentive, anticipating his optimal time to participate in REDD. Moreover, risk aversion postpones the optimal date at which the forester should optimally enter into the REDD scheme. This last finding justifies the design of REDD policies based on the transfer of market risk from the forester to an intermediary, e.g. a REDD project manager.

The paper is organized as follows: Section 2.2 gives an overview of the state of research in the combined fields of PES, PAM and environmental real options, at the crossroad of which lies our model. Section 2.3 introduces our setting and our model assumptions. Section 2.4 outlines the numerical method used to solve our model and the choice of parameters. Section 2.5 gives the main results and the key findings, followed by Section 2.5 which offers sensitivity analyses and a general discussion of our results. Section 2.6 concludes.

2.2 Literature Review

Apart from the decentralized financial element provided by the CDM market, projects on avoided deforestation have already been extensively tested under national development plans and important literature has emerged to document the complex, integrated and multi-layered processes driving deforestation. As underlined by Kaimowitz and Angelsen (1998) and Angelsen (2007), deforestation rates and harvesting patterns depend largely on a combination of local factors and global drivers: the degree of input/output markets integration, land tenure, access to capital and technological change, population pressure, migration and trade equilibria.

One of the first thoroughly documented performance assessments of a REDD prototype, the Noel Kempff climate action project in Bolivia (Grieg-Gran (2008) and Boyd (2009)), has reinforced the idea that reaching a win-win situation in a REDD project is a complex task embedding trade-offs between preservation, access to resources and efficiency of compensation. As noted by Asquith et al. (2002), *in the short run, certain sections of the local communities are financially poorer. Forest protection projects clearly have the potential to sequester [carbon], protect biodiversity and simultaneously contribute to sustainable rural development, but if they really are to improve rural livelihoods, they must be designed and implemented carefully and participatively.*

To that goal, REDD projects benefit from being currently at the junction of different research fields that deal with specific characteristics of the new avoided deforestation schemes.

The first characteristic is related to the economic estimates of avoided deforestation projects, taking into account spatial and economic variability. Grieg-Gran (2008) finds, as a result of a cross-country comparative analysis, that tropical forests represent in general an inexpensive way to mitigate climate change, with the notable exception of areas suitable for high values crops such as palm oil. Kindermann et al. (2008) conclude that the marginal costs are highly region-sensitive, with the lowest-cost region being Africa, followed by Central and South America and Southeast Asia.

The second line of research is focused on payments for environmental service (PES), which assess the incentive and performance aspects of compensation schemes. Through simulations based on a nationwide conservation program implemented in Mexican ejidos, Alix-Garcia et al. (2012) find that to ensure the maximum environmental benefits, flat payment should be replaced by conditional compensation schemes. Using observable results from the Costa Rican national PES scheme, Engel et al. (2009) conclude along the same lines that better PES targeting, using local

flexible payments and comprehensive assessment of the project's cost and threats, greatly improve its performance.
A third area of literature on protected area management (PAM) integrates conservation and development projects (ICDP), which have provided numerous insights into the complex combination of incentivizing policies deployed to ensure environmental preservation while extending local population's welfare. Muller and Albers (2004) evaluate the best mix of patrolling, agricultural project development and conservation payments for different market settings. They find that the efficiency of any policy instrument is highly dependent on the constraining nature of market access (whether it is complete market or missing labour/resources access). Robinson and Dupeyrat (2005) and Robinson et al. (2008) extend the PAM analysis to forestry and assess the implications of excluding rural people from newly protected forests.

An additional strand of research is the role played by market price uncertainty on deforestation schemes. Among the first to work on this question, Lembersky and Johnson (1975) introduce stochastic prices and inventory to define optimal harvesting. Miller and Voltaire (1983) extend their analysis to an ongoing rotation problem while Clarke and Reed (1990) define the optimal rotation problem in continuous time.
Using the seminal results from real options investment decisions, the forest management literature starts using optimal decision time in the presence of uncertainty and irreversibility to model forest extraction. Morck et al. (1989) use the contingent claims approach to determine the optimal rotation strategy under conditions of uncertainty. They assess the value of a forestry lease as the value of the option to cut down trees at the most advantageous time and allow for the possibility to halt production in case timber prices are too low. Thomson (1992) derives optimal forest rotation age when stumpage prices follow a diffusion process and employs the binomial model for the valuation of the standing forest. Plantinga (1998) shows analytically that reservation price policies can be very important in incorporating the option value of a forest when analysing optimal rotations, while Insley (2002) relies on the dynamic programming approach for characterizing the dynamics of the forest value, allowing for both geometric Brownian motion and mean reversion for the timber price process. An important recent contribution is the one of Alvarez and Koskela (2007), who analyse the partial and complete depletion harvesting policies under resource stock and price uncertainty and risk neutrality. They find that giving the manager the flexibility of partial sequential deforestation increases the value of the forest as compared with the case in which harvesting can be exercised only once. Moreover,

Monge et al. (2016) examine in a stochastic setting the trade-off between risk and returns of a forester/farmer. The authors find that, as risk aversion increases, a farmer would trade-off higher returns for a more certain stream of income. Finally, Pana and Gheyssens (2016), solving a dynamic model of land conversion from forest to agriculture in the presence of REDD, assess the performance of four baselines. They show that none of the assessed baselines dominates in all performance aspects, and that the final baseline choice needs to maximise the trade-off between the effectiveness to reduce deforestation, cost-efficiency, as well as changes in income.

While real options have been commonly used to analyse investment or divestment strategies, and while risk aversion has found in forestry a favourable field of study, the two have rarely been jointly applied in the literature to analyse the market for REDD permits, and the optimal investment strategy of a forester in terms of REDD projects. The impact of risk aversion is mitigated through the introduction of an intermediary REDD manager who bears the market risk of REDD, freeing the forester from it.

Analysing the impacts of risk aversion in forestry and land use change is also justified by the literature, such as Parks (1995), which finds that risk aversion is one important driver of deforestation and land use changes which are the practices that a REDD scheme aims to limit.

In order to establish how forest ecosystem services can be integrated into farming activities, Monge, Parker and Richardson examine in a stochastic setting the trade-off between risk and returns of a forester and how this influences his productive activities. They find that "a risk-averse farmer would trade off higher returns for a more steady stream of income". However, they do not provide indications as to how risk aversion can influence the intensity of farming, the amount of forest saved and the optimal timing of the forester/farmer's decision. In REDD projects, managing risk aversion is a key success factor as clearly stated by Blennow and Salinas (2006). One of the few works that relate real options and risk aversion is the one by Morel and Morel (2012), who propose a real option framework where the risk of a REDD project, solely aimed at reducing deforestation rates, is shared between the agents in the market "providing a per hectare payment that favours the conservation of forests containing higher biomass per hectare". However, the authors do not provide a price threshold triggering the option. We differentiate our research from the one of Morel and Morel (2012) first, by comparing how the forester's investment strategy changes when the risk perception of the REDD scheme changes; second, our model allows us to analyse which optimal investment threshold, in terms of REDD permit price, triggers the option to enter into the REDD scheme.

In synthesis, we contribute to the literature by developing a dynamic real options model capable of taking into account market failures and imperfect hedging in forest conservation decisions, examining the realistic setting where a forester has to bear part of the risk of the REDD project. This allows us to identify the optimal entry time into a REDD project, and to gage the impact of risk aversion on this optimum.

2.3 The Model

Our methodology relies on several extensions around a core first scenario which is presented in Section 2.3.2 and models the optimal choice of a risk neutral forester who has to decide when to optimally participate in a REDD scheme and what level of pre-REDD deforestation is optimal for him. An important improvement (Section 2.3.3) is the introduction of risk aversion for the forester, an individual behaviour that is common among local forest dwellers in developing countries and one which could have a strong impact on the effectiveness and efficiency of market-based REDD schemes.

2.3.1 A benchmark case: no REDD scheme

In the simplest scenario, a forester's decision is the determination of his constant extraction rate d, expressed in hectares of land deforested per unit of time, which he deforests for a given number of years τ, such that his intertemporal profits are maximized, under the simple constraint that the total deforested area through the extraction time can not be larger than his owned area of land λ. We can write the forester's maximization as

$$\max_{d} \int_{0}^{\tau} (P_f(t)d - c(d))e^{-\delta t}dt \tag{2.1}$$

under the constraint that

$$d \cdot \tau \leq \lambda \tag{2.2}$$

where $P_f(t)$ represents the timber price at each time t and which follows the deterministic setting:

$$dP_f(t) = \phi P_f(t)dt \tag{2.3}$$

with ϕ the growth rate of the timber price.

Following Cherian et al. (1999), we assume a quadratic harvesting cost function $c()$. This assumption allows for increasing marginal costs and the absence of costs for zero deforestation:

$$c(d(t)) = a_1 d(t) + a_2 d(t)^2 \tag{2.4}$$

with a_1 and a_2 the calibrated coefficients (see section 2.4). Under this simple setting, the forester has to account for the cost of extraction represented by the quadratic function $c(d)$, and the fractions of his land deforested at each point in time, modelled by the choice variable d.

2.3.2 A first case: a risk neutral forester deciding on a risky REDD project

Our initial model represents in stylized facts the behaviour of a risk-neutral forest manager who acts in a perfectly competitive market with fixed timber prices and the ability to voluntarily participate in a market REDD scheme at the time of his choosing (the REDD scheme is assumed to be immediately available).

At initial time, the forester owns a stock of forest λ that he can deforest at the fixed rate of d each time period. Each unit of forest preserved at the time of entrance into the REDD scheme is compensated by a rental value which is considered to be fully correlated with CDM spot permits (spot CER).

In mathematical terms, the forester faces the following inter-temporal profit maximization function:

$$\max_{L,d} E^{\mathbb{P}} \left[\int_0^{T_L} (P_f(t) \cdot d - c(d)) e^{-\delta t} dt + P_r(T_L) \left(\lambda - d \cdot T_L\right) e^{-\delta T_L} \right] \tag{2.5}$$

where

$$T_L = \inf\{t > 0 : P_r(t) \geq L\} \tag{2.6}$$

T_L represents the first time at which the forester voluntarily enters the REDD scheme (and irreversibly stops his deforestation activities) $\forall t \in T$; for further details please refer to Appendix A. $d(t) = d$ is the fixed annual deforestation flow as already explained in Section 2.3.1, while $c(d)$ represents the quadratic harvesting cost function as described by Eq. 2.4. L is the optimal exercise boundary (i.e. the price level at which it is optimal for the forester to enter the scheme instead of waiting longer). In our model, we assume the irreversibility of the REDD decision. Here $P_f(t)$ represents the timber price at each period t which follows the deterministic setting as described in Eq. 2.3, and $P_r(t)$ is the stochastic rental value of one hectare of preserved forest, which is perfectly correlated with the spot-CER price. We model the value dynamics as a geometric Brownian motion such that:

$$dP_r(t) = \alpha P_r(t)dt + \sigma P_r(t)dW_t$$

where α is the drift of the REDD price dynamics, σ their volatility and dW_t the increment of a Wiener process under the observable probability $\mathbb{P}$.

As previously mentioned, λ is the initial private share of forest at the forester's disposal. δ represents the discount rate.

In our model, we first assume that the extraction level is set at t_0 and remains fixed until the end of the deforestation sequence at T_L. This assumption is supported empirically by the existence of long-term trade agreements under which foresters engage in the fixed delivery of pre-agreed wood volume. Our setting would hence illustrate the forester's decision-making process before he enters into a new trade contract.

A second assumption of the model is the linear structure of REDD payoffs. Considering the latest development in the financial compensation structure of REDD schemes and past projects in project conservation, it seems clear that in reality foresters are only positively incentivized for avoided deforestation and not penalized if they don't succeed in curbing their deforestation levels.

In the context of a global forest parcelled out across multiple foresters, each have the possibility of participating in the REDD scheme. If one of them deforests more than his share λ, he encroaches another forester's area, dispossessing him of future REDD revenues. This would call for compensation for the forgone payments.

To solve for the different variations of our model, we rely on the real option nature of the decision

process (uncertainty and irreversibility) to inform a numerical grid search algorithm looking for the highest global profit on a very large interval of the decision variables.
In place of a naive algorithm, we use known analytical real option results to increase the speed of convergence towards the global solution. The optimization is derived from a martingale approach which simplifies several expectations within the maximization function.
First of all, by relying on the continuity of the brownian motion and on the linearity property of the expected value operator, we can recall the general optimization of the forester, as expressed by Equation 2.5, and where the dynamics of P_f are given by Eq. 2.3.
We start by solving the first integral of Expression 2.5:

$$\begin{aligned} & E^{\mathbb{P}}\left[\int_0^{T_L} (P_f(t)d - c(d))e^{-\delta t}dt\right] \\ = \;& P_f(0)d \cdot E\left[\int_0^{T_L} e^{-(\delta-\phi)t}dt\right] - c(d) \cdot E\left[\int_0^{T_L} e^{-\delta t}dt\right] \end{aligned} \tag{2.7}$$

Using the Laplace transform of the hitting time T_L, we can write:

$$E^{\mathbb{P}}\left[\int_0^{T_L} e^{-(\delta-\phi)t}dt\right] = \frac{1}{(\delta-\phi)}\left(1-\left(\frac{P_r(0)}{L}\right)^{\gamma_1}\right) \tag{2.8}$$

where

$$\gamma_1 = \frac{-\theta+\sqrt{\theta^2+2(\delta-\phi)}}{\sigma}, \qquad \text{with} \quad \theta = \frac{\alpha-\frac{\sigma^2}{2}}{\sigma} \quad \text{and} \quad \delta \geq \phi \tag{2.9}$$

According to the same lines:

$$E^{\mathbb{P}}\left[\int_0^{T_L} e^{-\delta t}dt\right] = \frac{1}{\delta}\left(1-\left(\frac{P_f(0)}{L}\right)^{\gamma_2}\right) \tag{2.10}$$

with:

$$\gamma_2 = \frac{-\theta+\sqrt{\theta^2+2\delta}}{\sigma} \tag{2.11}$$

The second part of Equation 2.5 can be solved as follows:

$$\begin{aligned} & E^{\mathbb{P}}\left[P_r(T_L)\,(\lambda - d\cdot T_L)\,e^{-\delta T_L}\right] \\ = \;& \lambda \cdot L \cdot \left(\frac{P_r(0)}{L}\right)^{\gamma_2} - d\cdot L \cdot \frac{\left(\frac{P_r(0)}{L}\right)^{\gamma_2} \ln\left(\frac{L}{P_r(0)}\right)}{\sigma\sqrt{\theta^2+2\delta}} \end{aligned} \tag{2.12}$$

where we relied on the fact that, at T_L, $P_r(T_L) = L$. For a detailed derivation please refer to Appendix B.

The forester's problem then reduces to:

$$\max_{L,d} \quad \Big[\frac{P_f(0)\cdot d}{(\delta-\phi)}\left(1-\left(\frac{P_r(0)}{L}\right)^{\gamma_1}\right)-\frac{c(d)}{\delta}\left(1-\left(\frac{P_r(0)}{L}\right)^{\gamma_2}\right)$$
$$+\lambda\cdot L\cdot\left(\frac{P_r(0)}{L}\right)^{\gamma_2}-d\cdot L\cdot\frac{\left(\frac{P_r(0)}{L}\right)^{\gamma_2}\ln\left(\frac{L}{P_r(0)}\right)}{\sigma\sqrt{\theta^2+2\delta}}\Big] \tag{2.13}$$

where γ_1 and γ_2 are defined, respectively, by Eq. 2.9 and Eq. 2.11, and θ by Equation 2.9.

In this scenario, the forester decides alone whether or not to join the REDD scheme and should fully internalize the trade-offs between the two activities.

Moreover, we initially assume that the forester is risk neutral[1]. Hence, he is solely interested in the maximization of his expected profits and not influenced by their volatility. While such a forester may very well exist, we consider this case to be a benchmark with which we will compare the decisions of the risk averse agents defined in section 2.3.3.

2.3.3 A second case: the introduction of risk aversion

It is well documented that most local foresters in developing countries are risk-averse, with a strong willingness to smooth the variations of their revenues. In such a context, a market-integrated REDD project poses the problem of being more risky (due to its stochastic price) than the deforestation revenues it seeks to replace. Therefore, the degree of risk aversion can be crucial to the decision to join and ultimately to the adhesion rate of REDD schemes in highly vulnerable contexts.

To model risk aversion, we use a classical CRRA utility function of the form (Szpiro, 1986):

$$U(x) = \frac{x^{1-\rho}}{1-\rho} \tag{2.14}$$

[1]We assume here that the risk neutrality comes from individual risk aversion behaviour and not from a fully diversified portfolio, which would prove unrealistic in our context.

with ρ the coefficient of relative risk aversion. The maximization problem of the forester then becomes:

$$\max_{L,d} E^{\mathbb{P}}[U(x)] \tag{2.15}$$

or, given Expression 2.5:

$$\max_{L,d} E^{\mathbb{P}} \left[\frac{\left(\int_0^{T_L} (P_f(t)d - c(d))e^{-\delta t} dt + P_r(T_L)\,(\lambda - d \cdot T_L)\, e^{-\delta T_L} \right)^{1-\rho}}{1-\rho} \right] \tag{2.16}$$

2.4 Model's parameters

The regions with the highest potential for developing REDD programs are Latin America (Amazon), Africa (RDC), and Indonesia (Bali, Borneo). Historically, the largest supplier of forest carbon credits is Latin America, being accountable for nearly 60% of the 2010 total primary market volume (Diaz et al., 2011). In the region, projects in Peru and Brazil dominate overwhelmingly.

In order to solve our model numerically, we calibrate it using observed data. We take into consideration a potential REDD project aimed at protecting the Peruvian forest from deforestation over a time frame of 100 years. The fact that Peru has one of the richest ecosystems in the world which includes 12 national parks and 63 remaining protected areas, makes this Latin American country the ideal candidate for our model. The parameters used are characteristic for Peru, as detailed in Table 2.1, and in most cases are taken from past cost/benefit analyses.

The data for the hypothetical Peru forest program comes from various official sources. For consistency of computational base, we convert the price of timber from $\$/m^3$ into $\$/ha$, by relying on the IPCC Good Practice Guide LULUCF (IPCC (2003)). The price of timber and its long term mean (ϕ) are obtained from the Peruvian market from the Annual Review and Assessment of the World Timber Situation (ITTO, 2010). The State of the Forest Carbon Markets 2011 (Diaz et al., 2011) is the source we use for the identification of the REDD permit price and its growth rate. The conversion of the deforested area into tons of carbon emitted is achieved with the help of another converter (Ω), whose value for Peru can be found in the OSIRIS model for the above and below ground biomass carbon and for soil carbon (OSIRIS). The discount rate used for comparing profits over time is 6%, a classical value also employed by Engel et al. (2012). For the calibration of the cost of timber extraction, we adapt the cost function of Angelsen (1997), calibrating it to data from Verissimo et al. (1992) for the Amazon rainforest.

Table 2.1: Model Calibration Parameters

Parameter	Explanation	Value	Sensitivity Analysis	Source
$P_f(0)$	Timber price	500 $\$/m^3$	-	ITTO (2010)
ϕ	Timber growth rate (annual)	0.015	-	ITTO (2010)
C	$\$/m^3$ to $\$/ha$	158 m^3/ha	-	IPCC (2003)
$P_r(0)$	REDD permit price (annual, in $\$/tCO_2$)	5 $\$/tCO_2$		Forest Trend (2011)
$P_r(0)$	REDD permit price (annual, in $\$/m^3$)	21 $\$/m^3$	[1; 500]	Forest Trend (2011)
α	REDD permit price drift (annual)	0.04	-	Forest Trend (2011)
σ	REDD permit price vol. (annual)	0.1	[0; 0.3]	Forest Trend (2011)
Ω	ha to tC emitted	179 tC/ha	-	OSIRIS (v3.4)
ψ	tC to tCO_2	3.67 tC/tCO_2	-	Assante (2011)
δ	Discount rate	0.06	[0; 0.1]	-
d	Deforestation Flow	-	[0; 5]	
a_1	Cost parameter	3.3198 $\$/ha$	-	Angelsen (1997), Verissimo et al. (1992)
a_2	Cost parameter	798.0811$\$/ha$	-	Angelsen (1997), Verissimo et al. (1992)
ρ	Coefficient of relative risk aversion	0.5	[0; 1]	Chronopoulos et al. (2011)
λ	Forest area under own.	200	-	-

2.5 Results and Sensitivity Analysis

Based on the chosen parameters, a first important result is the forester's optimal decision in a risk neutral setting. When profits solely come from timber extraction under the specific discount rate δ, growth rate of timber price ϕ and quadratic cost function $c(d)$, the risk neutral forester optimally cuts all his endowed area λ in a sequence of 5.5 years, each year with a deforested area of $d = 36.4$ hectares. In this setting, the forester's optimal decision implies no protection for his forest, as he will never be incentivized to preserve it by entering into a REDD scheme. This depends mainly on one factor, i.e., the deforestation rate d. The question then to be posed is what his optimal choice would be in case he is forced to keep his deforestation rate within a certain upper boundary, i.e. 1 hectare per year. This limitation make sense as it ensures that the incentive to protect part of the forest is kept until the end of the investment period, which, in our case, is 100 years. Results for this scenario can be seen in Table 2.2.

Table 2.2: Optimization Results, $0 \leq d \leq 1$, Risk Neutral Investor

Optimal Deforestation Flow ($\bar{d}$)	Optimal Investment Threshold ($\bar{L}$)	Expected First Passage Time ($E[T_{\bar{L}}]$)
$d = 1$	155\$/m^3	year 57
Total Area Deforested	**REDD Revenues ($\hat{R}$)**	**Number of Permits Sold (P_{sold})**
57 ha (28.5% of λ)	22,165\$	143 (1 per m^3)

We can see that the forester still deforests at the maximum rate. However, in contrast to what happens in the first scenario, he now has an incentive to participate in the REDD scheme. To be more precise, he will enter it at year 57.

2.5.1 Sensitivity Analysis

How does the forester's optimal choice change when risk increases, here denoted by the volatility parameter σ? Intuitively, as risk increases, the forester will wait longer before entering the REDD scheme. In other words, his optimal investment threshold increases. Figure 2.1 illustrates this result which must be compared with initial REDD price $P_r(0)$ of 21 $\$/m^3$.

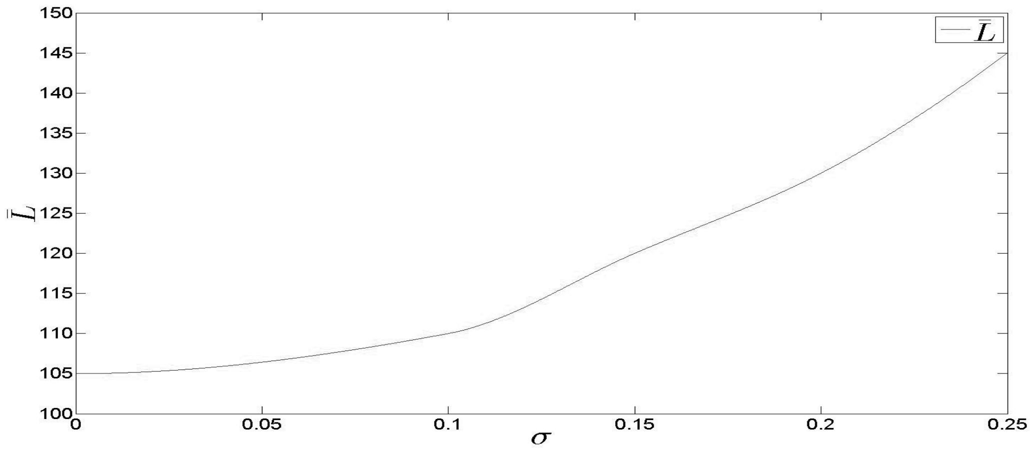


Figure 2.1: Optimal Investment Threshold $\bar{L}$ vs. Volatility σ, for $\delta = 6\%$

Increasing the discount rate from 6% to 10% decreases the optimal investment threshold $\bar{L}$, consequently anticipating the entry time in the REDD program as well, as shown in Figure 2.2. This has interesting implications as a higher discount rate makes early entry into a REDD project more profitable for the forester. As in the previous example, these results have to be compared to an initial REDD price $P_r(0)$ of 21 $\$/m^3$.

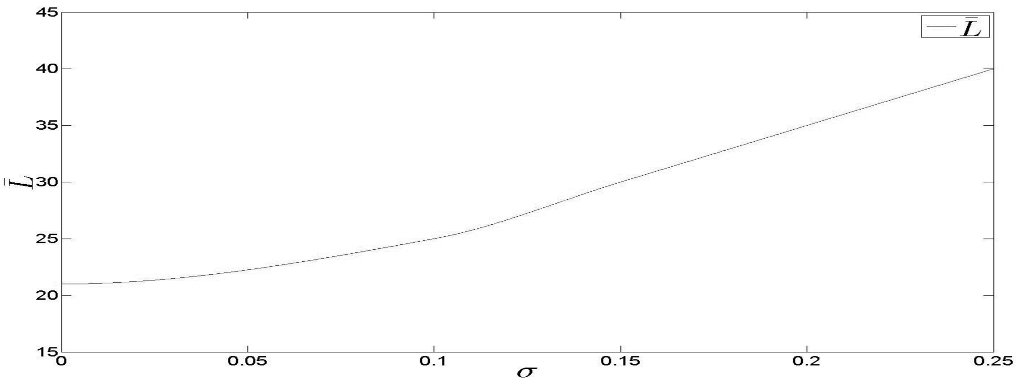
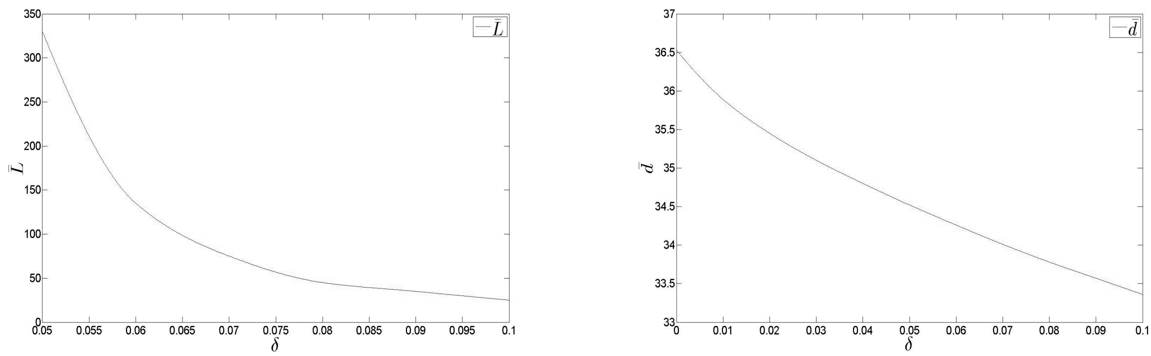


Figure 2.2: Optimal Investment Threshold $\bar{L}$ vs. Volatility σ, for $\delta = 10\%$

An increase in the discount rate δ not only lowers the optimal investment threshold $\bar{L}$, but also the optimal deforestation rate $\bar{d}$. While this may seem surprising, the fact that higher discount rates imply lower future timber value causes the optimal REDD entry time to shift towards earlier years. An additional effect is a decrease in the deforestation rate since less forest is cut and more can be protected under REDD. These results are illustrated in Figure 2.3a and Figure 2.3b.



(a) **(b)**

Figure 2.3: Optimal Investment Threshold $\bar{L}$ (Figure 2.3a) and Optimal Deforestation Rate $\bar{d}$ (Figure 2.3b), plotted vs. Discount Rate δ

The effect of volatility on the expected first hitting time can be seen in Figure 2.4. In line with traditional real options theory, the forester prefers to wait longer before committing to a REDD scheme when volatility is high.

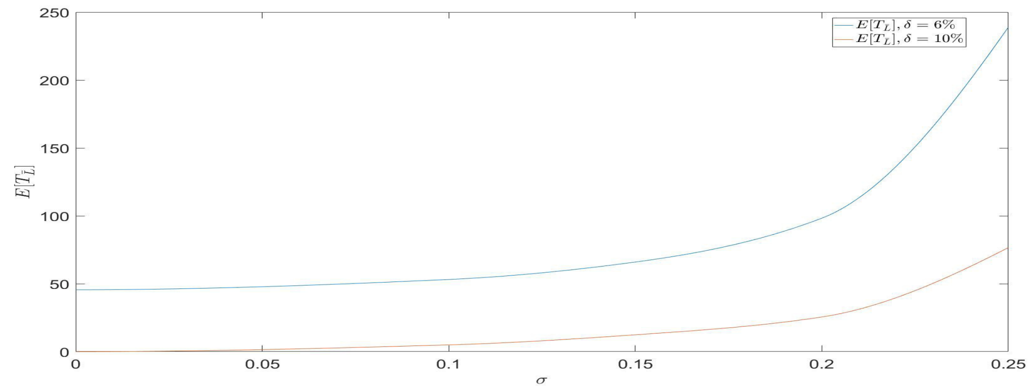


Figure 2.4: Expected First Passage Time vs. Volatility σ, for $\delta = 6\%$ and for $\delta = 10\%$

Volatility increases the optimal deforestation rate as well. Indeed, as risk increases the forester prefers to deforest more and obtain higher profits from selling the timber given the higher uncertainty about future REDD prices. This behaviour is derived from the fact that, by doing so, he can enter into the REDD scheme with a good revenue base. While he might obtain higher profits from REDD, the increased uncertainty makes him more cautious. This is shown in Figure 2.5.

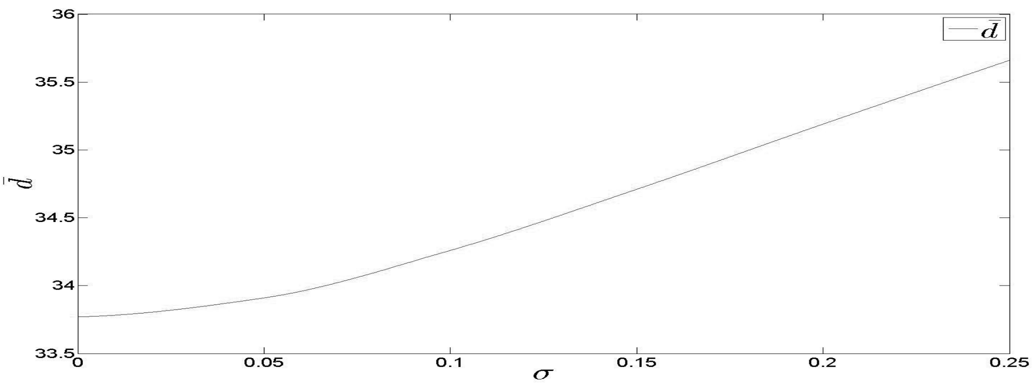


Figure 2.5: Optimal Deforestation Rate $\bar{d}$ vs. Volatility σ

2.5.2 A Risk Averse Forester

Introducing risk aversion partially changes the picture. Indeed, a risk averse forester enters the REDD scheme at later dates compared to a risk neutral one. The reason is clear. As REDD permits bear inside market and liquidity risk, a risk averse investor will want to wait longer and obtain more information about the riskiness of the project before irreversibly committing to it. Figure 2.6 illustrates this behaviour for a volatility of $\sigma = 0.1$ and a discount rate of $\delta = 10\%$.

This result justifies the presence of intermediation in REDD markets. An intermediary, i.e. a REDD manager, can provide the forester with a deterministic stream of payments in exchange for forest protection, while bearing the risks of such markets. This would function as an incentive for a risk averse forester to participate in a REDD scheme earlier, thus preserving more forest from being cut.

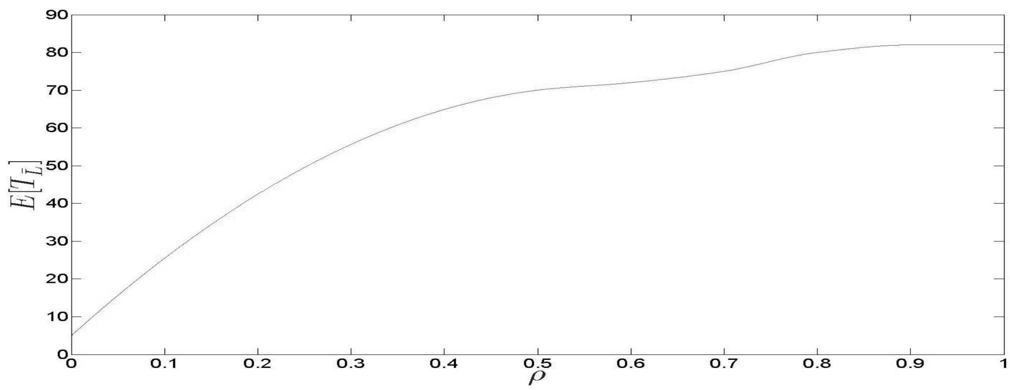


Figure 2.6: Expected First Passage Time vs. Coefficient of Relative Risk Aversion ρ

2.6 Conclusion

Implementing a Real Options Approach, we have been able to assess a forester's investment decision when choosing between deforesting or protecting his forest, committing his land to a Reducing Emissions from Deforestation and Forest Degradation (REDD) scheme. The choice is important and has strong implications for the forester. Entering a REDD scheme is an irreversible decision, that provides the forester with uncertain, stochastic, cash flows instead of a certain, deterministic, revenue. In order to be as realistic as possible, our model takes into consideration different risk profiles of the forester. Indeed, we test our model with a risk neutral investor, and with a risk adverse investor.

Results show that the value of a risk neutral forester's option to enter into a REDD scheme, thereby protecting his forest from deforestation, but also renouncing the cash flow provided by the sale of the timber, is maximized when the deforestation rate d is low. However, this is only true when authorities oblige the forester to cut within certain deforestation boundaries. If such boundaries are removed, the forester is incentivized to cut the entire forest within a few years. Moreover, we find that lower discount rates postpone the decision to participate in a REDD scheme, while higher ones, as is the case in many developing countries, prove to be an incentive, anticipating his optimal time to enter REDD. In fact, higher discount rates lead to a lower value for cash flows in the future, causing the optimal REDD entry time to shift towards earlier years. The introduction of risk aversion offers a more realistic perspective of the model and has the following consequences: it postpones the optimal date at which the forester should enter the REDD scheme and increases the annual optimal amount of timber to be cut, thus causing additional deforestation. Our analytical findings are in line with the ones of Monge et al. (2016) that examine in a stochastic setting the trade-off between risk and returns of a forester/farmer, showing that, as risk aversion increases, a risk-averse farmer would trade off higher returns for a more certain stream of income, which results in harvesting more forest more intensively. We contribute by showing that as the risk aversion coefficient increases, not only is the timber harvested more intensively, but the optimal entry time into a REDD project is increasingly postponed as well.

Furthermore, we investigate the extent to which REDD price uncertainty influences the decision to enter the scheme. Rising price volatility increases the optimal deforestation rate and postpones optimal entry into the REDD scheme. Indeed, as price uncertainty increases, the forester prefers to deforest more and obtain higher profits from selling the timber. This behaviour stems from the fact that, by doing so, he can enter the REDD scheme with a good revenue base. While he might obtain higher profits from REDD, the increased uncertainty makes him more cautious. Moreover, lower discount rates postpone the decision to enter a REDD scheme, while higher ones, as is the case in many developing countries, prove to be an incentive, anticipating his optimal time to enter REDD.

These results justify the introduction of a REDD manager into the market, who would bear the risks of a REDD permit market while providing the forester with a deterministic rent. In this case, in fact, thanks to the presence of an *intermediary* in the market, the forester would not face any risk and would again be incentivized to participate in the REDD scheme, therefore saving his forest from deforestation. This behaviour serves as an indication for policymakers on the importance of intermediation in such risky markets, as they have been struggling with the design of a functioning market for REDD that provides its agents with the right incentives to enter and trade while hedging their risks. This argumentation is supported by important literature such as Blennow and Salinas (2006), who argue that managing risk aversion is a key success factor in REDD projects.

However, additional research is needed. In particular, different utility functions could be applied to our model to test alternative forester preferences. In addition, interactions between the forester and a REDD manager could be analysed in order to check for the existence of principal-agent issues that may hinder the scheme from reaching its goals.

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Appendix A

The Hitting Time

$$
\begin{aligned}
T_L &= \inf\{t \geq 0 : P_r(t) \geq L\} = \inf\{t \geq 0 : P_r(0)e^{(\alpha-\frac{\sigma^2}{2})t+\sigma W_t} \geq L\} = \\
&= \inf\{t \geq 0 : e^{(\alpha-\frac{\sigma^2}{2})t+\sigma W_t} \geq \frac{L}{P_r(0)}\} = \\
&= \inf\{t \geq 0 : \gamma t + W_t \geq \frac{1}{\sigma} ln\left(\frac{L}{P_r(0)}\right)\} = \\
&= \inf\{t \geq 0 : Z_t \geq l\} \qquad \text{(A.1)}
\end{aligned}
$$

where $\gamma = \frac{(\alpha-\frac{\sigma^2}{2})}{\sigma}$, $l = \frac{1}{\sigma} ln\left(\frac{L}{P_r(0)}\right)$ and $Z_t = \{\gamma t + W_t, t \geq 0\}$ a $\mathbb{Q}-$Brownian Motion, drifted under the probability measure $\mathbb{P}$.Troja (2016) Chesney et al. (2016)

Appendix B

Expression 2.5

$$\begin{aligned}
& E^{\mathbb{P}}\left[P_r(T_L)\,(\lambda - d\cdot T_L)\,e^{-\delta T_L}\right] \\
= \; & E^{\mathbb{P}}\left[\lambda P_r(T_L)e^{-\delta T_L} - d\cdot T_L\cdot P_r(T_L)e^{-\delta T_L}\right] \\
= \; & \lambda\cdot\underbrace{E^{\mathbb{P}}\left[P_r(T_L)e^{-\delta T_L}\right]}_{\text{Expression } 2.5a} - d\cdot\underbrace{E^{\mathbb{P}}\left[T_L\cdot P_r(T_L)e^{-\delta T_L}\right]}_{\text{Expression } 2.5b}
\end{aligned} \tag{B.1}$$

B.1 Expression 2.5a

$$E^{\mathbb{P}}\left[P_r(T_L)e^{-\delta T_L}\right] = L\cdot E^{\mathbb{P}}\left[e^{-\delta T_L}\right] = L\cdot\left(\frac{P_r(0)}{L}\right)^{\gamma_2} \tag{B.2}$$

where:

$$\gamma_2 = \frac{-\theta+\sqrt{\theta^2+2\delta}}{\sigma}, \qquad \text{with} \quad \theta = \frac{\alpha-\frac{\sigma^2}{2}}{\sigma} \quad \text{and} \quad \alpha\leq\delta \tag{B.3}$$

B.2 Expression 2.5b

$$
\begin{aligned}
& E^{\mathbb{P}}\left[T_L \cdot P_r(T_L)e^{-\delta T_L}\right] = L \cdot E^{\mathbb{P}}\left(T_L \cdot e^{-\delta T_L}\right) = L \cdot \frac{\partial}{\partial \delta} E^{\mathbb{P}}\left(-e^{-\delta T_L}\right) \\
= \quad & L \cdot \left[\frac{-\partial}{\partial \delta}\left(\frac{P_r(0)}{L}\right)^{\gamma_2}\right] = L \cdot \frac{\left(\frac{P_r(0)}{L}\right)^{\gamma_2} \ln\left(\frac{L}{P_r(0)}\right)}{\sigma\sqrt{\theta^2 + 2\delta}}
\end{aligned} \tag{B.4}
$$

where:

$$
\gamma_2 = \frac{-\theta + \sqrt{\theta^2 + 2\delta}}{\sigma}, \qquad \text{with} \quad \theta = \frac{\alpha - \frac{\sigma^2}{2}}{\sigma} \tag{B.5}
$$

Appendix C

Published Manuscript: ***Mitigating Global Warming: A Real Options Approach***

Ann Oper Res
DOI 10.1007/s10479-016-2258-5

S.I.: ENERGY AND CLIMATE POLICY MODELING

Mitigating global warming: a real options approach

Marc Chesney[1] · Pierre Lasserre[2] · Bruno Troja[1]

Abstract Mitigation and adaptation represent two solutions to the issue of global warming. While mitigation aims at reducing CO_2 emissions and preventing climate change, adaptation encompasses a broad scope of techniques used to reduce the impacts of climate change once they have occurred. Both have direct costs on a country's gross domestic product, but costs also arise from temperature increases due to inaction. This paper introduces a tipping point in a real options model and analyzes optimal investment choices in mitigation and their timing.

Keywords Adaptation · Mitigation · Real options · Delay · Tipping point · Climate change · CO_2 · Gross domestic product

1 Introduction

Climate change has become increasingly important in political discussions. The Intergovernmental Panel for Climate Change (IPCC) has expressed strong concerns about the eventual consequences for the planet and humanity if mean temperatures reach or increase above the 2 °C threshold. Since this temperature increase seems inevitable at this point, given the CO_2 emission trend of past years, the IPCC is calling for rapid efforts to prevent further warming, via *mitigation*, and to reduce the effects of already rising temperatures on natural and social systems, via *adaptation* (IPCC 2014). Indeed, climate change has huge potential negative effects. Lower food supplies, water shortages, droughts, and increased health problems are among the consequences of high CO_2 concentrations that negatively influence production and consumption, which in turn impact current and future economic growth. The situation is already more critical than expected, and negative feedback effects are imminent (IPCC 2014).

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Other potential effects are catastrophic, both in terms of system dynamics and in the common meaning of the world. The potential collapse of the Atlantic thermohaline circulation and its effects on the lives of millions of people is a clear example of such abrupt changes (Huber and Knutti 2012).

To take these potential effects into account, this article models the impact of climate change and the possible occurrence of a catastrophic event on global welfare. The possibility of catastrophic events is widely acknowledged in the literature, and their implications have been investigated at both theoretical and empirical levels (see next section). Catastrophic events occur when the state of the climate reaches a tipping point—the threshold—with strong feedbacks that trigger one or several events. Such catastrophic events could include the interruption of the thermohaline circulation, massive methane releases, or the melting of ice caps causing a rise in sea level. In this paper, we define a catastrophic event as an irreversible disruption having a dramatic negative impact on humanity. If the catastrophic sequence of events is triggered, even a return to pre-industrial conditions will only allow the ice caps or the methane sinks to reconstitute themselves over such a long period that their loss may be considered irreversible for the purposes of human society. The change in climate regime and the new conditions prevailing over the planet will thus be established irreversibly. Although the human species would not be wiped out, the costs would be high enough -and the subsequent conditions of human activity uncertain enough- that it is justified to model this catastrophe as a long-lasting collapse of the gross domestic product (GDP) and as an interruption of the dynamic optimization problem addressed by our model for the period preceding the catastrophe. We assume that the climate state defining a tipping point can be modelled as an atmospheric temperature threshold. Our paper further considers an aspect neglected in climate and economic modelling: catastrophes are likely to be triggered only if temperatures stay above some threshold for a certain time (Lenton et al. 2008). This time window has to be given particular attention. Short periods above the temperature threshold would not lead to any drastic departure from the continuous pattern of damage associated with temperature while a long period above the threshold temperature would trigger a catastrophe.

The above assumptions imply that the catastrophe is certain not to happen in the immediate future as long as temperature stays below some threshold level. However, the likelihood of a catastrophe occurring within a given future period increases as temperature rises, since the rise means that the threshold becomes more likely to be reached and also exceeded by the process for the duration of the *time window*. Furthermore, a long-lasting business-as-usual policy will lead to a catastrophe. Consequently, the decision maker must monitor the temperature process and decide whether or not to devote resources to slow down or reverse the rise in temperature. This is the mitigation decision. Mitigation has been studied in a number of ways that we discuss briefly in the next section. We model it as a once-and-for-all irreversible decision to start spending some endogenous proportion of GDP on it after some optimally chosen temperature level has been reached. This determines a reduction in emissions and thus a modification of the temperature process, which is stochastic in our setting. While this is a typical real options setup, its solution is not conventional and involves a methodological contribution outlined in the text and precisely described in the "Appendix".

Adaptation is different from mitigation. First mitigation is a pure global public good while adaptation involves actions that are either private or whose public dimension is much less pronounced. The decision by an individual to move to safer grounds is largely private. Protective dikes are public goods, but only locally, and public institutions deal much better locally than globally for the provision of public goods. With a climate treaty, for example, free rider problems appear. For this reason, we treat adaptation as exogenously determined within the model, without any intervention of the decision maker, while we treat mitigation as a planning decision. The decision maker optimizes mitigation for an economy whose GDP already incorporates the consequences of decentralized adaptation.

The second important difference between adaptation and mitigation is that adaptation does not affect the temperature process that determines climate. As a result, adaptation has no effect on the probability of occurrence of a catastrophe. We further assume that, when the catastrophe occurs, adaptation measures taken prior to the event are without effect on the consequences of the catastrophe, which is that GDP equals zero as of this date. This is because the damages are different in nature from those resulting from a progressive change in climate and are also much more difficult to envisage. Consequently, we treat adaptation as a decentralized activity affecting welfare before the possible climate catastrophe but without any impact on its consequences. Given the path of GDP, net of the impact of adaptation, the decision maker optimizes the additional welfare impact of mitigation while considering its effect on the probability of climatic catastrophe.

The questions that our research is trying to answer are the following: (1) what is the optimal percentage of GDP, net of adaptation expenditures, that a global decision maker should invest in climate change mitigation efforts? (2) when should such investment start, i.e. what is the optimal mean temperature that should trigger mitigation? (3) must mitigation expenditures be higher than those for adaptation, or vice versa? And finally, (4) how do investments in mitigation affect the probability of occurrence of the catastrophic event? We will provide detailed answers to these questions in Sect. 5.

The article is structured as follows. The current state of the literature on GDP impacts of climate change will be discussed in Sect. 2 while the model will be presented and explained in detail in Sect. 3. Section 4 will present the dynamic optimization of the model, Sect. 5 will show the numerical results obtained, and Section 6 will draw some conclusions.

2 Literature review

A number of issues on climate change are addressed in the economic literature. These include the cost of climate change, the potential for mitigation and adaptation, and the instruments that must be mobilized as well as the timing of action. The impacts are identified in terms of growth in GDP, food supply, or the stock of man-made or natural capital. Empirical assessments differ widely, but there is a broad consensus that impacts are unevenly distributed across world regions.[1] Another area of consensus is that climate change is an immense challenge

[1] A huge literature investigating the economic impacts of climate change and the need for mitigation or adaptation measures has focused on specific areas and sectors, in particular on agriculture and the future availability of food supplies. Fischer et al. (2005) studied the interactions between climate change and different development paths. They suggest that climate change will worsen the gap between developed and less developed countries. For Fischer et al. (2005), adaptation in agricultural techniques is the key to limiting the impacts of climate change on crops. Rosenzweig and Parry (1994) showed that adaptation at a local-farm level is insufficient. Action, in the form of mitigation, is needed at a global level. Similar conclusions were drawn by Parry et al. (2004) who considered different socio-economic scenarios under the Intergovernmental Panel on Climate Change, Special Report on Emissions Scenarios (Nakićenović and IPCC, Working Group III. 2000). While their results depend heavily on the effects of CO_2 concentration on agriculture yields, which are unknown, Parry et al. (2004) infer that the world will be nevertheless able to feed itself since the diminished production in developing countries will probably be counterbalanced by an increased production in developed countries. However, this does not justify inaction; inequalities at regional and local levels may become socially and economically devastating. Fankhauser (1997) estimated costs and benefits of climate change and how these impact economies in the former Soviet Union, China, the United States of America, the European Union, other OECD countries, and other non-OECD countries. He found that climate change is likely to cause a loss of 1.5 % of the world GDP.

for economic institutions. First, and despite a tempering note by Battaglini et al. (2014), this is because climate change is the biggest instance of the tragedy of the commons ever recorded (Stern 2007; Stavins 2011). As such, it cannot be addressed without some interference with the decentralized operation of markets. Second, climate change is the first instance of the tragedy of the commons occurring at a truly global scale. It is not likely to be solved by the methods that societies have developed at local and regional levels to deal with similar problems at smaller scales. A theoretical literature initiated by Barrett (see, e.g., Barrett 2005, 2013) analyzes the difficulties involved in reaching international agreements in that context. As a result, much of the literature is normative, and our paper also falls into this category.

A substantial part of the economic climate change literature consists of integrated assessment models (IAM). Although some have the appearance of positive analyses, their conclusions are invariably used to fuel debates over normative issues. Economic models of climate change and their outcomes have been investigated by Nordhaus and Boyer (2003) and Tol (2002a). Nordhaus and Boyer (2003) developed a model, called RICE for Regional Integrated model of Climate and Economy, which is an improvement of the famous DICE model (dynamic integrated model of climate and economy; (Nordhaus 1992).[2]

One of the most important studies about the effects of climate change on world GDP is that of (Stern 2007). This author calculates the monetary impacts of inaction (or insufficient action) on the global economy. Stern (2007) found that due to inaction, the world may lose up to 5 % of its aggregate GDP each year. If all the possible risks are taken into account, as in a worst-case scenario, costs on world GDP could add to 20 % or even more. These costs are very high compared to what the author calculated as the amount needed to combat climate change, i.e., about 1 % of world GDP if carbon has to be stabilized at around 550 ppm. Similar results, though slightly less striking, are found in IEA (2006). A recent study by Fundacion DARA Internacional and Climate Vulnerable Forum (2012) on the monetary impacts of climate change on world GDP found that about 3.2 % of world GDP by 2030 (or 1.2 trillion a year) are at risk because of climate change and because of the inaction of governments around the world. The Stern review relies on various information and methodologies, especially those of IAMs.

IAMs have been harshly criticized for their lack of objectivity and transparency in policy applications.[3] Pindyck (2015) and others argue in favour of simple pedagogical models able to enlighten decisions but that certainly must not be relied on for accurate answers. A variety of models may claim to fall into that category. A brief review not only shows their variety and richness, but helps identify and justify the climatic and economic features that we choose to emphasize in this paper. Golosov et al. (2014) developed a dynamic stochastic general equilibrium (DSGE) model that allows the identification of the optimal carbon tax,

[2] The RICE model is able to predict the economic impacts of climate change in different regions. Like other important literature, results show that developed regions would, on average, profit from an increase in global mean temperatures, while the impact on developing countries would be the opposite. However, the two effects are not of the same magnitude, since climate change affects the poorest areas with much more devastating outcomes compared to what richer communities would experience. Tol (2002a) finds that the impacts of changing climate on GDP are extremely model-dependent, since they can be positive, negative, or non-existing according to how prices are taken into account. In general, however, it is evident how impacts have different consequences depending on the country or group of countries under analysis, whether developed (OECD, Middle East, China) or non OECD. Similar results were obtained by Tol (2002b), where dynamic estimates were introduced.

[3] Pindyck (2015) argues that "...Because the modeller has so much freedom in choosing functional forms, parameter values, and other inputs, the model can be used to obtain almost any result one desires, and thereby legitimize what is essentially a subjective opinion about climate policy."

or, equivalently, the marginal externality damage of emissions.[4] Bretschger and Vinogradova (2014) model an economy in which global warming causes stochastic climate shocks that negatively impact the capital stock. They found an optimal flow of emission abatement that is able to reduce climate shocks.[5] Mitigation and adaptation represent the core of the analysis by Bahn et al. (2012). These authors found that investments in mitigation highly depend on the effectiveness of adaptation measures; effective adaptation may reduce or completely suppress the need for mitigation.[6]

Pindyck (2015) claims that "what really matters [for the social cost of climate change] is the likelihood and possible impact of a catastrophic climate outcome: a much larger-than-expected temperature increase and/or a much larger-than-expected reduction in GDP caused by even a moderate temperature increase. IAMs, however, simply cannot account for catastrophic outcomes". Prieur et al. (2011) and Amigues and Moreaux (2013) introduced a threshold catastrophic temperature as the key element of an economic climate change model where the catastrophe causes infinitely large damage. While they used a dynamic but non-stochastic framework, Tsur and Zemel (2008) also consider the possibility of a catastrophic climate event in a stochastic environment. However, the random occurrence of the catastrophe is not directly linked to temperature or CO_2 thresholds; it is governed by a Poisson law, where the parameter increases with cumulative emissions. While they differ in their treatment of uncertainty, the above papers highlight the importance of catastrophes.

In this paper, we model a climate catastrophe as an irreversible event of such magnitude and with such manifestations that it amounts to the end of society as we knew it before the catastrophe, with no basis to conceive of the ensuing area. We model this as calling an end to the optimization period. Although this is by design an extreme representation of a climate catastrophe, it is not without scientific basis. Dakos et al. (2008) and Lenton et al. (2008) find that a deviation from a threshold temperature sustained over time is capable of inducing dramatic changes to the environment. Lenton et al. (2008) identified several policy-relevant tipping elements, i.e., events or climate states that could keep the temperature process above a certain threshold for a long time window.

As a matter of fact, researchers differ widely in the way they have modelled catastrophes and their consequences. Baranzini et al. (2003) model an environmental catastrophe incorporating negative jumps in the stochastic process corresponding to the net benefits associated with abatement policies. Lemoine and Traeger (2014) investigated the welfare costs of a tipping point, finding that a sufficiently high carbon tax is necessary to mitigate abrupt climate shifts and that such a tax is capable of reducing peak temperatures by as much as 0.5 °C. Following the set-up of Naevdal (2006), Naevdal and Oppenheimer (2007) deal with the trigger of an environmental catastrophe, i.e., the interruption of thermohaline circulation, which

[4] The authors find that the damage is proportional to the current GDP and that the degree of proportionality is only dependent on the discount rate, on the elasticity of damage, and on the structure of carbon depreciation in the atmosphere. Interestingly, important elements such as consumption, population dynamics, technological paths, and CO_2 concentration in the atmosphere have no influence on the damage caused by emissions. In addition, they find that the optimal carbon tax should be higher than the median tax estimated by the literature.

[5] The optimal flow spent on abatement exhibits a constant growth rate and is an increasing function of the intensity of the environmental damage. They suggest that a world with uncertainty requires more stringent climate policies that a world without.

[6] However, if a catastrophe is to occur with some probability when a threshold temperature is reached and its occurrence is not affected by adaptation but only by mitigation, then adaptation increases the probability of such catastrophe as it takes resources away from mitigation. Prevention can also be suboptimal due to uncertainty about the future (de Bruin and Dellink 2011). However, while preventive adaptation may include strong delays before being effective, as happens with mitigation (Bahn et al. 2012), reactive adaptation reduces uncertainty and delivers results more quickly, as pointed out by Parry et al. (2009).

would occur if "the temperature or rate of temperature change exceed certain [unknown] thresholds". The authors distinguished two unknown thresholds that trigger the collapse of thermohaline circulation. One is related to the rate of temperature increase and one is related to the temperature level itself.[7] Similarly, Keller et al. (2004) studied the effects of an unknown threshold that causes the interruption of thermohaline circulation. These optimal stopping models are similar in that respect to the real option model presented in this paper.[8] Weitzman (2007) found the probability of crossing a threshold temperature level higher than 8 °C relative to pre-industrial level to be approximately 3–4 %; the negative consequences of such a catastrophe are impossible to appraise, whether qualitatively or in magnitude.

Our model is strongly inspired by the literature on tipping points triggering catastrophic natural events. That is, the catastrophe is certain not to happen as long as the threshold is not reached, in contrast to models where a catastrophe is possible with some probability whatever the state (as when its occurrence obeys a Poisson law). Similar to Amigues and Moreaux (2013), we model the catastrophe as a dramatic event of such magnitude that there is no need or possibility to model—let alone manage—the post-catastrophe regime, as in Lemoine and Traeger (2014) and the controlled IAMs that they discuss. We do so in a stochastic environment. To avoid unrealistic outcomes, where the catastrophe occurs with certainty as soon as a known threshold is reached, many authors have assumed that the threshold is unknown, implying that learning about the threshold may occur, i.e., if some state is reached and no catastrophe occurs, one knows that the threshold must be higher. The economy is then safe if it remains at or below the state already reached.

The setting in this paper differs in that the (temperature) threshold is assumed to be known, but the catastrophe occurs only if enough time passes above that threshold. Since the process is stochastic, the catastrophe is uncertain even when the threshold is exceeded, but society is obviously taking risks if it allows that to happen. We believe that this is a more realistic way to model the scientific evidence described in Lenton et al. (2008) than assuming that the catastrophe is certain once the threshold is reached. Global temperature levels are a good example, since a prolonged period above a certain temperature level is needed for a natural catastrophe such as the interruption of thermohaline circulation or melting of the Western Antarctic Ice Sheet to occur.

Perhaps the most important characteristic that distinguishes economic from other climate models is when the former seek to optimize some policy variable.[9] In that respect, remarks such as those of Weyant (2008) about the Stern review—that climate policy should not be taken as a deterministic "one shot" benefit-cost analysis but as a problem of sequential decision-making under uncertainty- are to be taken seriously. Nonetheless, "one shot", or

[7] Naevdal and Oppenheimer (2007) found that the upper boundary of the rate of temperature increase is crossed in finite time while the upper boundary of the temperature process is crossed only as time goes to infinity.

[8] The authors found that increased uncertainty does not increase optimal abatement. The reason can be found in one (or both) of these conditions: (a) risk aversion is not the dominant nonlinearity in their model and (b) increased uncertainty does not decrease the variance of the per capita consumption. Thus, characteristics of the threshold and the learning process have a strong influence on the optimal abatement policy in the near-term. Similar results in a setting with an unknown (but reversible) catastrophic threshold are also analyzed by Brozović and Schlenker (2011), who found a non-monotonic relationship between precautionary behaviour and uncertainty. Higher uncertainty surrounding the natural system usually increases precautionary behaviour. However, when the risk becomes large enough, the behaviour of the decision maker becomes less precautionary because "precautionary reductions in pollutant will be too costly compared to the negligible expected reduction in the probability that the threshold is crossed".

[9] Examples are the optimal carbon tax in the stochastic dynamic general equilibrium model of Golosov et al. (2014), the model with tipping points of Lemoine and Traeger (2014), or the optimal flow of carbon abatement in Bretschger and Vinogradova (2014).

infrequent, decisions on policy variables are the rule more often than the exception in climate policy. The real option approach deals with such timing decisions.

In finance, an option is a title that gives its owner the right, but not the obligation, to buy (in the case of a call option), or to sell (a put option) another financial title such as a stock. If and when the option is exercised, there is no return to the previous situation. A real option involves a similar decision (in the case of a call option, buy irreversibly, or abstain from buying), except that the decision applies to a 'real' thing rather than to financial paper. For example the owner of a project such as a road may exercise the option to build the road by spending resources to that end; once the resources have been committed, there is no return or 'deconstruction' of the road. For many such projects, there is an optimal time to undertake them, to exercise the option.

The environmental real option approach is based on the premise that, and applies when, environmental policies involve committing resources for the long term and are irreversible due to institutional and other constraints. Under such conditions, environmental policies are best modelled as once-and-for-all (or long-term) decisions (Pindyck 2000; Insley 2002; Kassar and Lasserre 2004) whose timing must be chosen. This is perhaps most obvious if we think that climate problems may have to be solved by treaties (Barrett 2013). In such cases, and many others, environmental policy decisions are costly to reach and to characterize, so they take the form of a once and for all policy decision that is irreversible and requires dedicated resources, whether they are dollars or political capital. These "one shot" decisions could be compared to the results of the COP 21 in Paris in December 2015. In the current model, instead of fixing an objective in terms of temperature increase until the end of the century, the central planner decides when and how much to invest. The timing of the policy and the magnitude of the engagement must be chosen optimally.

Real option models are particularly well adapted to such optimal stopping problems. They are used in order to check whether or not investment decisions should be taken. The standard tool used in this setting before real options were introduced is the Net Present Value (NPV in what follows) approach according to which, an investment should be realized if and only if its NPV, i.e. the difference between its expected discounted payoffs and costs, is positive. This latter criteria is static to the extent to which the choice is between realizing the investment at the date when the NPV is calculated, or never.

This is a significant drawback of the NPV criterion. If investment opportunities are considered as real options, the investor has the right, and not the obligation, to make an investment during a given period of time. When identifying the optimal investment date, the possibility of postponing the investment is taken into account. See Dixit and Pindyck (1994) for an excellent reference. Usually, the timing of the investment is the objective of these models. In this paper, not only the timing of the investment in mitigation is obtained, but also the optimal amount to be invested.

In fact the concept of option value was introduced in environmental economics before the appearance of real options (Arrow and Fisher 1974; Henry 1974; Fisher and Krutilla 1975). It stresses that making an irreversible action at one point in time involves the cost of renouncing the flexibility to wait; if this cost is correctly taken into account in a cost benefit analysis the benefits from the decision must be higher than in a traditional cost benefit analysis for the action to be economically justified. In terms of a climate change decision, if the benefits from curbing emissions are higher, the higher the temperature reached by the planet, then the irreversibility of the decision to control emissions would raise the temperature threshold at which the decision to curb emissions is implemented. Moreover, this threshold would be higher, the more volatile the temperature process.[10]

However, the 'traditional' view that the relationship between volatility and threshold is monotonic has been challenged. As described by Pindyck (2000), real option decisions may involve two kinds of irreversibilities that work in opposite directions. First, an environmental policy imposes sunk costs on society, and political constraints may make the policy itself difficult to reverse. Second, environmental damage can be partially or totally irreversible. For example, increases in GHG concentrations are long lasting, and the damage to ecosystems from higher global temperatures can be permanent. In other words, while there is a direct cost of exercising the option which can be avoided by postponing the exercise, there is also a cost incurred while waiting. Thus, adopting a policy now rather than waiting has a sunk benefit -a negative opportunity cost- that biases traditional cost-benefit analysis against policy adoption.

Our model examines a similar trade-off. We assume that the decision maker knows the tipping point temperature and that he has to make an optimal decision, in terms of when to invest in mitigation, by choosing an optimal temperature threshold, and in terms of how much to invest, by choosing an optimal fraction of GDP to devote to mitigation.

3 The model

The average global surface temperature (Hasselmann 1976; Kaerner 1996; Lawrence and Ruzmaikin 1998; Eby et al. 2009, 2012) and world GDP dynamics (Brock and Mirman 1972) can be respectively modelled by a time component plus a random part, driven by white noise and a volatility parameter, i.e.,

$$dC_t = \begin{cases} a dt + \beta dW_t & \text{for} \quad t < T_L + \Delta T(k, L) \\ a(k)dt + \beta dW_t & \text{else} \end{cases} \tag{1}$$

and

$$\frac{dV_t}{V_t} = \mu dt + \sigma dB_t \tag{2}$$

where $\{W_t, t \geq 0\}$ and $\{B_t, t \geq 0\}$ are two independent Brownian motions[11] under the physical probability $\mathbb{P}$ and where the drift parameters, a and μ, and the volatility parameters, β and σ, are constant and positive. In particular, $a > 0$, the drift of the temperature process, explicitly models the global warming effect we are experiencing today. We will also assume that the discount rate r is constant and positive. It is worthwhile mentioning that there is no guarantee that the temperature process C will remain positive with probability one. However, given our initial set of parameters, it is highly likely. The GDP process V, expressed in dollars, is by definition positive.

The temperature process evolves in two phases, a "before mitigation" and an "after mitigation" phase beginning at time T_L, as soon as the threshold L is reached. The impact of such a mitigation strategy on the temperature process will start with a delay, i.e. , at time $T_L + \Delta T(k, L)$. There is also an autonomous GDP process and a *net* GDP process; the latter is a function of both temperature excess from the pre-industrial temperature, $C_P = 14\,°\mathrm{C}$

[10] On the effect of volatility on threshold values in basic real option models, see Dixit and Pindyck (1994), especially chapters 1 and 5.

[11] In an arithmetic Brownian motion setting, the drift a and the volatility β are both expressed in degrees Celsius.

and the autonomous GDP process V_t. To be more specific, climate change causes a flow of day-to-day costs over time, and these costs can be viewed as levies from GDP as time goes by. Human adaptation efforts can reduce their immediate impacts to some extent, but not suppress them. Consequently, we introduce the disposable GDP, $DGDP_t$, as the GDP, V_t, net of the day-to-day costs of climate change as moderated by adaptation efforts:

$$DGDP_t = V_t e^{-\rho|C_t - C_P|} \tag{3}$$

where $C_P = 14\,^{\circ}\mathrm{C}$ describes the global average temperature level prior to industrialization, in the absence of man-made pollution, and where $\rho > 0$ is a parameter reflecting the impact of the temperature gap and its measurement units. Note that this functional form implies strong convexity with respect to $C_t - C_P$, meaning that the effect of inaction is accentuated if the temperature process C and the temperature level prior to industrialization, C_P, diverge. The higher the difference between C_t and C_P, the more accentuated its impact on adaptation costs and therefore on the disposable GDP.

A global environmental catastrophe will occur if the temperature remains without interruption above a given temperature level L_1 over a period of D units of time. This specification finds its justification in a vast literature on tipping points and abrupt climate change, which is reviewed with a focus on policy implications by Lenton et al. (2008). Abrupt climate change occurs "when the climate system is forced to cross some threshold, triggering a transition to a new state at a rate determined by the climate system itself and faster than the cause" (p. 1786). In fact, deviations above the tipping point L_1, sustained over time (for D units of time), are capable of creating serious negative effects on the environment. According to Hansen et al. (2008),

> Paleoclimate data and ongoing global changes indicate that 'slow' climate feedback processes not included in most climate models, such as ice sheet disintegration, vegetation migration, and GHG release from soils, tundra or ocean sediments, may begin to come into play on time scales as short as centuries or less. (p. 217)

Indeed, as these authors argue, if the overshoot of the appropriate long-run CO_2 target is not brief, there is a high probability of seeding irreversible catastrophic effects.[12] The catastrophe will therefore not occur the first time that the temperature reaches the critical level L_1, but only if it remains above this critical level without interruption for a given period of time. The full impact of global warming on possible climate catastrophes therefore requires a given time window. As soon as the catastrophe occurs, the GDP is approximated by zero and is assumed to remain at this level as of this date. This is a specific feature of this model as compared with others. The real options setting allows the joint determination of the optimal temperature level L^* at which mitigation should start to be implemented and the optimal level k^* of this investment. Optimality means that these two values are endogenously specified in order to maximize the expected sum of discounted GDPs between the current time and the date of the catastrophe.[13]

[12] Hansen et al. (2008) argue that 'If humanity wishes to preserve a planet similar to that on which civilization developed and to which life on Earth is adapted, paleoclimate evidence and ongoing climate change suggest that CO_2 will need to be reduced from its current 385 ppm to at most 350 ppm.(...) If the present overshoot of this target CO_2 is not brief, there is a possibility of seeding irreversible catastrophic effects.' While the target of 350 ppm is low when compared with other targets that are considered reasonable, in particular some proposed by the IPCC, the idea that a long overshoot will trigger a catastrophe that might have not occurred with a brief incursion in the non-sustainable zone, appears very reasonable.

[13] Similar situations have been studied in finance with Parisian options; the delay is called Parisian time. For the mathematical specification of Parisian time and other variables, please refer to "Appendix 1".

3.1 Adaptation and mitigation

Adaptation is modelled as the magnitude of the adjustment in GDP implied by the constant ρ in Eq. (3). The higher ρ the bigger the net day-to-day impact of climate change on disposable GDP. A value of $\rho = 0$ means that there is no impact of temperature change on GDP, hence no adaptation efforts, so that $DGDP_t = V_t$. When ρ is strictly positive, disposable GDP would depart from V_t as the temperature gap $C_t - C_P$ rises. As temperature increases, an increasing proportion of GDP is lost to the day-to-day costs of climate change.

Adaptation efforts range from entirely private (changing residence) to partially public (building levies for local protection), as opposed to mitigation, which is a pure public good at the global scale. It is thus reasonable to assume that ρ is determined by market mechanisms and local institutions that function efficiently, whereas mitigation decisions have to be studied as a decision maker's problem. Adaptation efforts have no effect on climate dynamics, they only affect the way current temperature translates into current disposable GDP, and they only do so while climate dynamics remain in the current climate regime. If temperature rises to such a level that the climate dynamic system undergoes some catastrophic change, previous adaptation efforts will not have any impact on the magnitude of the catastrophe.

Mitigation is modelled via two endogenous variables: first the flow of investment k, measured as a proportion of GDP, aiming at decreasing the drift of the temperature process (1); second the optimal date T_L[14] at which mitigation expenditures are introduced. An entity, such as an international organization, chooses to devote a proportion $ke^{-\delta(s-T_L)}$, with $s \geq T_L$, of world disposable GDP to activities or measures that reduce the rate of increase of the temperature process relative to some business-as-usual trajectory. This proportion may be constant if $\delta = 0$ or, if $\delta > 0$, it diminishes exponentially from its maximum k that occurs at the date T_L on which the mitigation decision is taken. When $\delta < 0$, then too much time has passed with inaction, and a mitigation effort increasing over time is needed.

Given the difficulties surrounding the mitigation decision process, the decision to slow down the process driving climate change should be viewed as being reached very rarely -we assume once at most- and as irreversible. For example, it may be interpreted in the model as a treaty whose features would be respected once the treaty is signed, although there might be a delay until these features are fully implemented and a delay until their effect is felt. For example, consider a decision that is implemented when the global temperature reaches some endogenous threshold level L, at date T_L. If a fraction $ke^{-\delta(t-T_L)}$ is spent for mitigation as of T_L, then the temperature process will be *modified* only after a given delay $\Delta T(k, L)$, i.e., the trend of the temperature will be set to $a(k)$ instead of remaining at a at time $T_L + \Delta T(k, L)$. The delay is defined as

$$\Delta T(k, L) = \frac{\theta}{V_{T_L} e^{-\rho(L-C_P)} k} \tag{4}$$

where T_L represents the date at which the decision to mitigate is taken, i.e., the first passage time of the temperature process at level L, and θ is a parameter that models the delay magnitude, which can be influenced by chemical and atmospheric factors, and also by elements such as type of mitigation, whose choice is not modelled here.

The delay $\Delta T(k, L)$ takes into account the fact that the higher the starting disposable GDP available for expenditures in mitigation and the higher the fraction of disposable GDP actually spent, k, the quicker the effect on the temperature process. In addition, the wider the temperature gap $L - C_P$ at the time the mitigation decision is implemented, the longer

[14] As shown later, T_L is the random time at which the temperature process reaches a predetermined level L, which triggers the flow of investments k in mitigation.

the delay because the day-to-day costs of climate change, despite adaptation efforts, use up a portion of the disposable income otherwise available for mitigation. The function of $\Delta T(k, L)$ finds its rationale in important scientific literature (Friedlingstein et al. 2011), which showed that, despite a sudden drop in carbon emissions and due to strong inertia, it still takes (much) time for the temperature process to stabilize and eventually start decreasing. For additional information, please refer to "Appendix 4".

In addition to the delay required for mitigation expenditures to become effective, their initial size k, as a proportion of disposable capital, determines their impact on the temperature trend, which they reduce from a to $a\ (k)$ according to the formula[15]

$$a(k) = a - (a - \eta)\frac{k}{\alpha} \tag{5}$$

with

$$\lim_{k \to \alpha} a(k) = \eta \tag{6}$$

where η is a negative constant reflecting the self-regenerating capacity of the atmosphere, expressed here in terms of its effect on the temperature process; its determination will be described shortly.

Equation 6 indicates that when mitigation efforts are set to a proportion $k = \alpha \leq 1$ of $DGDP$, then the drift of the temperature process reaches η so that anthropogenic effects are eliminated and the self-regenerating capacity of the atmosphere becomes the sole non-stochastic factor affecting temperature changes. In other words, $\alpha \cdot 100\,\%$ of the GDP should be spent in order to eliminate these anthropogenic effects. Note that in this model, an $k = a/(a-\eta) < \alpha$ is required in order to stabilize the temperature level ($a(k) = 0$). While an $\alpha = 1$ corresponds to a pessimistic scenario, it is an indisputable ceiling that corresponds to our choice in this paper. Obviously, a smaller value of α could be used in our model. We will illustrate this possibility in Sect. 4.

Equation 5 can be rationalized as follows. Assume that Eq. 1 is an approximation of

$$dC_t = (\eta C_t + f(N) + f(E_t))\, dt + \beta dW_t \tag{7}$$

where $f(\cdot)$ represents a function that models the impacts of natural emissions N and of anthropogenic emissions E_t on the temperature process per unit of time, and ηC_t is the drop in temperature induced by the gross (before emissions) self regeneration of the atmosphere, when temperature is C_t; η is a negative parameter to be determined. $f(N)$ is assumed to remain constant over the industrial period while $f(E_t)$ is null at the beginning of industrialization and positive thereafter. It is assumed that the drift of the temperature process was zero before the industrial period, since natural emissions on average offset self regeneration; hence

$$f(N) = -\eta C_P \tag{8}$$

where $C_P = 14.0\,°\mathrm{C}$ represents the average global pre-industrial temperature. Substituting into Eq. (7) yields:

$$dC_t = (\eta\,(C_t - C_P) + f(E_t))\, dt + \beta dW_t. \tag{9}$$

Since $\eta < 0$, Eq. (9) models the dynamics of a mean reverting process. In particular, if $C_t = C_P$, the drift reduces to $f(E_t)$.

[15] Since a is independent of δ at this stage, we will write $a(k)$ instead of a of $a(k, \delta)$.

In this paper, Eq. (1) is used instead of the complex Eq. (9) to model the temperature process. The drift a corresponds to:

$$a \cong \eta\,(C_0 - C_P) + f(E_0). \tag{10}$$

The negative parameter η may be approximated as follows. The self-regenerating capacity of the atmosphere is often defined as the natural rate of resorption of the CO_2 stock (Hansen et al. 2008; Archer et al. 2009); estimates vary widely. The assumption that the natural rate of resorption of the CO_2 stock is constant and that 25 % of emitted CO_2 is still in the atmosphere after two centuries (Friedlingstein et al. 2011) implies a decay rate of 0.1 % per year. In terms of temperature process, we recall that the atmosphere was in stationary equilibrium at a temperature of $C_P = 14.0\,°\mathrm{C}$ during the pre-industrial period. Suppose that this equilibrium is disturbed by the sudden emission of a quantity of carbon that instantaneously raises temperatures by one degree. That carbon will still be in the atmosphere after two centuries, so that the gap in temperature away from the stationary equilibrium will vanish at the rate of 0.1 % per year because of the natural decay of CO_2. Thus the influence of natural carbon decay on temperature is $0.001\,(C_t - C_P)$ and

$$\eta = -0.1\,\%. \tag{11}$$

At dates before $T_L + \Delta T(k, L)$, the dynamics of the temperature process are given by (1). By spending a proportion $ke^{-\delta(t-T_L)}$ of disposable GDP, as of T_L it is possible to reduce anthropogenic emissions, thus reducing the drift of the temperature process from a to $a\,(k)$ as of $T_L + \Delta T(k, L)$. Although it is theoretically possible to achieve negative emissions by carbon sequestration techniques, we assume that the maximum possible reduction, obtained by starting with a proportion $k = \alpha$ at T_L, is to reduce anthropogenic emissions to zero, as in the pre-industrial state. On the other hand, if $k = 0$, nothing is changed, so $a\,(0) = a$. Equation 5 expresses this relationship.

Thus, the choice of the fraction k affects both the new drift and the delay until the new drift applies. Since a substantial portion of mitigation expenditures takes the form of investments into R&D and technologies for mitigation, we may think of the proportion $ke^{-\delta(s-T_L)}$ of disposable GDP set aside for mitigation as ensuring that the capital necessary to maintain the temperature drift $a\,(k)$ is built up and maintained. This may require higher initial efforts, followed but a somewhat reduced capital maintenance effort. This possibility is crudely modelled by parameter δ, as explained earlier.

The associated drift of C_t thus changes at the endogenous time $T_L + \Delta T(k, L)$, from a to $a(k)$. The temperature process' dynamics thus change from Eq. (1) to

$$d\hat{C}_t = a(k)dt + \beta dW_t, \qquad \text{for} \quad t \geq T_L + \Delta T(k, L) \tag{12}$$

To this will correspond a *new* disposable GDP

$$\widehat{DGDP}_t = V_t e^{-\rho|\hat{C}_t - C_P|}, \qquad \text{for} \quad t \geq T_L + \Delta T(k, L) \tag{13}$$

which has the same form as the disposable GDP described in Eq. (3), but which is now a function of a temperature process C_t with a different drift.

Let us note T_L as the first passage time of the temperature at a level L that triggers the decision to use the budget k allocated with the mitigation

$$T_L = \inf\{t \geq 0 : C_t \geq L\} = \inf\{t \geq 0 : Z_t \geq l\}.$$

with $\{Z_t = \gamma t + W_t, t \geq 0\}$ a drifted $\mathbb{P}$–Brownian Motion, $l = \frac{L-C_0}{\beta}$ and $\gamma = \frac{a}{\beta}$. The mitigation budget $V_s e^{-\rho(C_s - C_P)} ke^{-\delta(s-T_L)}$ will be spent as a continuous flow from T_L

to the date of the catastrophe, denoted $H^{+}_{L_1,D}$. This is the first time that the temperature process remains without interruption above the temperature level L_1 for D units of time. Its mathematical definition is given in "Appendix 1".

3.2 The objective function

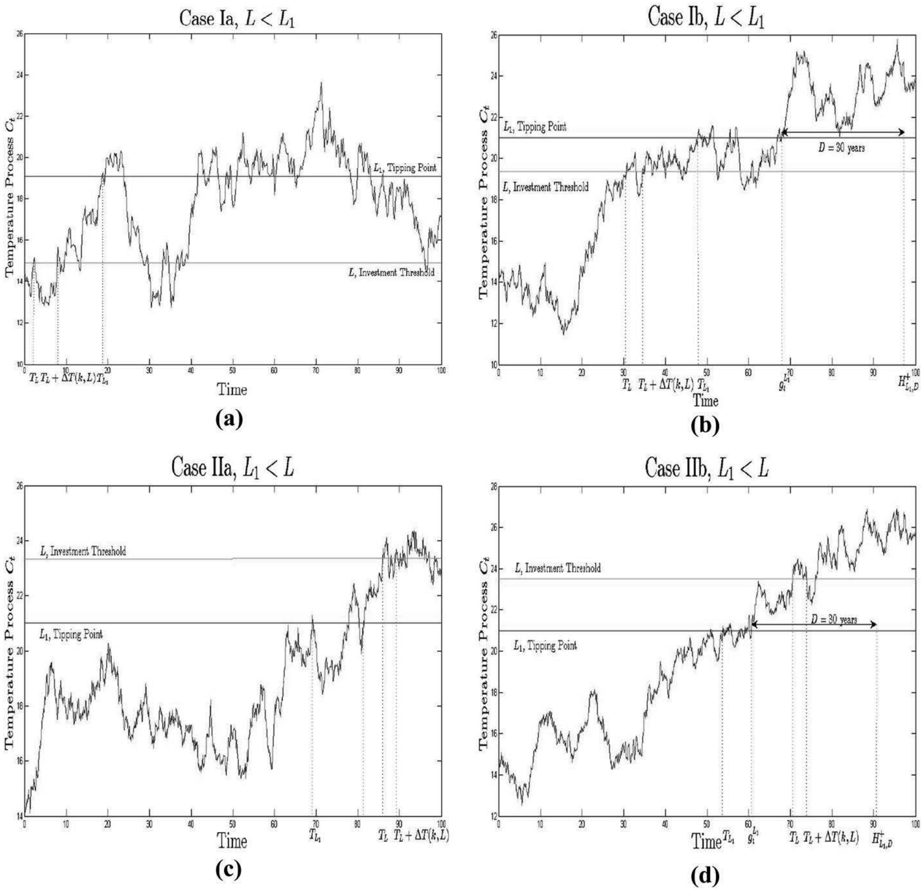
We are taking the point of view of a global decision maker who maximizes the discounted cumulative future disposable GDP over the next $T = 500$ years by choosing two variables. He selects a threshold temperature L that triggers the beginning of the mitigation investment period;[16] he also determines the magnitude of the investment by choosing the proportion k of disposable GDP devoted to mitigation from the beginning of the mitigation period. The decision to undertake mitigation causes the drift of the temperature process to drop from its historical level a to a lower level $a(k)$ after a delay $\Delta T(k, L)$. The choices of L and k have no effect on the tipping level L_1; however, they affect the date at which L_1 may be reached as well as the probability of a catastrophe, i.e., the probability that temperature stays above L_1 continuously for at least D years.

Let us now consider the following cases for L.

1. $\mathbf{L < L_1}$—i.e., the endogenous threshold L which triggers the investment in mitigation, is lower than the exogenous threshold L_1 that may trigger catastrophic events. As long as T_L, the first passage time of temperature at a level L, is smaller than the horizon T, he will invest a fraction k of his budget in mitigation at time T_L, causing the temperature process to lower its drift. The decrease in the drift does not happen immediately after the investment is made, but is subject to a time delay equal to $\Delta T(k, L)$. After $T_L + \Delta T(k, L)$[17], and if the temperature process has not yet reached the tipping point level L_1, the temperature process is then both less likely to reach L_1 and less likely to stay above L_1 for a long period of time than in the absence of the mitigation decision. Here, two different things can happen. In fact, it can be that $T_L + \Delta T(k, L)$ is small enough to avoid a catastrophe (Case Ia). Figure 1a illustrates this situation, i.e., the one in which the investment in mitigation has been promptly made. This has caused a decrease in the temperature drift already at early stages, thus avoiding the catastrophe. Conversely, it could happen (Case Ib) that the time $T_L + \Delta T(k, L)$ is not small enough to avoid a catastrophe, as illustrated by Fig. 1b.
2. $\mathbf{L > L_1}$—i.e., the endogenous threshold L which triggers the investment in mitigation, is higher than the exogenous threshold L_1 that may trigger catastrophic events. If the global decision maker decides to invest at a temperature higher than the level L_1 that triggers the catastrophe, he faces two possible situations. In the first one (Case IIa), it could happen that L, the threshold temperature that triggers an investment k in mitigation, is reached before a possible catastrophe. If this happens, then the global decision maker finds himself in a situation similar to Case I, where mitigation expenditures may still be sufficient to avoid the catastrophe before time T, through a timely decrease in temperatures; however, the reduction in drift will need to drive the temperature process below L_1 before D units of time are spent consecutively above the threshold, which is of course less likely than if the threshold had not been reached in the first place, as in Cases Ia and Ib. It can also happen (Case IIb) that D units of time are spent continuously above the catastrophe level

[16] The choice of a determinate optimal temperature level determines the optimal time at which the mitigation policy should be implemented; however that time remains random because the date at which any given temperature is reached is random.

[17] Note that time $T_L + \Delta T(k, L)$ might be shorter or longer than time T_{L_1}.



(a)

(b)

(c)

(d)

Fig. 1 Simulation of one path. The *green line* represents the level L at which the global social decision maker starts investing in mitigation. The *solid red line* represents the level L_1, which is the temperature above which the catastrophe can be triggered, if the process stays continuously above L_1 for a period of time at least equal to D

L_1. In this case, even if a mitigation procedure is brought forward, the environmental catastrophe occurs in a finite period. These two possibilities are illustrated in Fig. 1c, d.

The four different cases pictured in Fig. 1 imply different formulations of the objective function presented below. The global decision maker determines an optimal investment threshold L^* and an optimal investment fraction k^* such that the expected discounted sum of the future disposable GDP is maximized. In order to do so, he has to find the supremum, over L and k, of a function $f(\cdot, \cdot)$. Because the horizon can be considered infinite, it is known in the options literature that the optimal trigger level L^* is constant (Merton 1973; Carr et al. 1992; Chesney and Jeanblanc 2004).

The maximization problem simplifies to

$$\sup_{k,L} f(k, L) \Leftrightarrow \sup_{k,L} \left[\mathbf{1}_{L<L_1} \cdot g_1(k, L) + \mathbf{1}_{L \geq L_1} \cdot g_2(k, L) \right] \tag{14}$$

with:

$$g_1(k, L) = E_{\mathbb{P}} \Bigg[\underbrace{\int_0^{T_L \wedge T} DGDP_u e^{-ru} du}_{I_1 a} + \underbrace{\int_{T_L \wedge T}^{T_L + \Delta T(k,L) \wedge H^+_{L_1,D} \wedge T} DGDP_u \left(1 - k e^{-\delta u}\right) e^{-ru} du}_{I_1 b} + \underbrace{\int_{T_L + \Delta T(k,L) \wedge H^+_{L_1,D} \wedge T}^{H^+_{L_1,D} \wedge T} \widehat{DGDP}_u \left(1 - k e^{-\delta u}\right) e^{-ru} du}_{I_1 c} \Bigg] \tag{15}$$

and

$$g_2(k, L) = E_{\mathbb{P}} \Bigg[\underbrace{\int_0^{T_L \wedge H^+_{L_1,D} \wedge T} DGDP_u e^{-ru} du}_{I_2 a} + \underbrace{\int_{T_L \wedge H^+_{L_1,D} \wedge T}^{T_L + \Delta T(k,L) \wedge H^+_{L_1,D} \wedge T} DGDP_u \left(1 - k e^{-\delta u}\right) e^{-ru} du}_{I_2 b} + \underbrace{\int_{T_L + \Delta T(k,L) \wedge H^+_{L_1,D} \wedge T}^{H^+_{L_1,D} \wedge T} \widehat{DGDP}_u \left(1 - k e^{-\delta u}\right) e^{-ru} du}_{I_2 c} \Bigg] \tag{16}$$

This is a generalized optimal stopping problem with two stochastic processes, one associated with the temperature process and one associated with the GDP process. As already mentioned the choice of optimal stopping rather than optimal control as methodology for addressing the climate change problem is due to the irreversibility of the investment decision into mitigation. Assuming that control variables are chosen at each period, as in stochastic control problems, is simply not realistic in the climate change framework.

We also argued that real option models are particularly well adapted to optimal stopping problems. They are used in order to check whether or not investment decisions should be taken; the price at which they should be taken is called the *strike price*. In the real options model presented here, what plays the role of the *strike price* are the integrals (preceded by a minus sign) in the terms labelled $I_1 b$, $I_1 c$, $I_2 b$, and $I_2 c$ of Expressions 15 and 16. They correspond to mitigation costs.

The integral bounds in Expressions 15 and 16 are in most cases stopping times of the temperature process C and the functions to be integrated involve the two stochastic processes, C and V, that determine both welfare before a possible catastrophe, and the likelihood of that catastrophe.

The objective function can be intuitively interpreted in the following way:

- If $\mathbf{L} < \mathbf{L_1}$, i.e. if the endogenous threshold L which triggers the investment in mitigation is lower than the exogenous *tipping level* L_1 that may trigger catastrophic events:

  1a. Integral I_1a computes the expected discounted sum of future disposable GDPs from the initial date until time T_L, when the decision to invest in mitigation is taken by the global social decision maker, or until time $T = 500$ years, whichever comes first;

  1b. From time T_L until time $T_L + \Delta T(k, L)$, without catastrophe in the meantime, an investment for mitigation has been made, but it is still too early for the drift of the temperature process to shift down. This will happen only at time $T_L + \Delta T(k, L)$ and if the catastrophe has not occurred. The expected discounted sum of future disposable GDPs is reduced by the fraction k invested in mitigation (Integral I_1b);

  1c. From time $T_L + \Delta T(k, L)$, until time $H^+_{L_1,D}$ when the catastrophe happens, or until time T, the drift of the temperature process is lower. If $H^+_{L_1,D}$ is higher than T, Integral I_1c computes the expected discounted sum of future disposable GDPs until time $T = 500$ years; if the temperature process stays above L_1 for at least D years without interruption then the catastrophe occurs and future GDPs becomes null as of $H^+_{L_1,D}$.

- If $\mathbf{L} > \mathbf{L_1}$, i.e., if the threshold that triggers the investment in mitigation is higher than the exogenous *tipping level* L_1 that may trigger catastrophic events:

  2a Integral I_2a represents the cumulative GDP from the initial date until whichever time happens first: a) time T_L, when the decision to invest is taken, b) $H^+_{L_1,D}$, when the catastrophic event happens, or c) $T = 500$ years. If either b or c is the case, then all the following integrals are zero, since no investment in mitigation (Integral I_2b) or change in temperature drift (Integral I_2c) occurs.

  2b If T_L occurs later than $H^+_{L_1,D}$, Integral I_2b is zero. Otherwise, this integral corresponds to I_1b.

  2c Integral I_2c computes the expected discounted sum of future disposable GDPs during the period of time that goes from $T_L + \Delta T(k, L)$ to $H^+_{L_1,D}$, the time when the catastrophe occurs, or to T. Integral I_2c is strictly positive when $H^+_{L_1,D}$ is greater than $T_L + \Delta T(k, L)$ and zero otherwise.

The formulation chosen to represent the second part of the objective function allows us to set Integral I_2c equal to zero if $H^+_{L_1,D}$ is reached after T_L, but before $T_L + \Delta T(k, L)$. When $H^+_{L_1,D} < T = 500$ is smaller than T_L, this formulation assigns zero values to both Integral I_2b and Integral I_2c.

From a mathematical point of view, by referring to Section 2.1 of Peskir and Shiryaev (2006), it can be stated that the problem is well-defined and well-posed. Indeed, as shown in "Appendix 2", the expectation of the supremum over time of the profits associated to the real option's exercise is bounded.

If the horizon is finite, as it is the case when the model is implemented, then depending on the parameters, the stopping time might exist. If it exists, it will by construction be smaller than horizon T. If it does not exist, it means that the central planner will finally decide that it is not optimal to start a mitigation policy. For example, if the discount rate is too high the decision maker might delay this investment and then notice that it is too late in any case. This case is not ex-ante excluded in the model and is a possible result.

If the horizon is infinite, depending on the parameters, a solution might exist, as in the previous case. If the stopping time exists, then the issue of its unicity is resolved as follows. The first stopping time will be selected. The logic of the model is as follows. At current time t, there are two possibilities. Either the optimal stopping time has been reached and the mitigation policy should be implemented without delay, or otherwise the model should be subsequently used as long as the stopping time has not been reached. If it is reached in a finite time, then the investment should start at that date and the optimal GDP percentage k^* to be invested is the one defined at this same date.

As shown empirically in Fig. 9a, b in "Appendix 3", in the specific case where the temperature process is deterministic, the function to be maximized behaves quite nicely and exhibits a unique maximum for the optimal temperature level at which the mitigation strategy should be implemented L^*, and for the optimal amount to be invested k^*.

If the stopping time is never reached it means that it is infinite and the mitigation policy will never be implemented. If at initial time t, with the current set of parameters, it is not optimal to implement the mitigation policy, the question of the existence of the stopping time remains open. It might occur at a finite time in the future, or be infinite.

4 Calibration and numerical results

The starting point of the numerical simulations is the year 2011, and $C_0 = 14.8\,°\mathrm{C}$ (287.95°K) is considered as the baseline temperature. The time horizon chosen is $T = 500$ years, with timesteps Δt of 1 year. Parameter values for the Monte Carlo simulation are given in Table 1 and are discussed below.

The catastrophe threshold L_1 was chosen as 19 °C, a temperature at which environmental events that could reshape the livability of Earth, such as the interruption of the thermohaline circulation (Bahn et al. 2011; Lenton et al. 2008), can happen with positive probability. These values are in line with IPCC (2013), which estimated a possible temperature increase in the range 1.1–6.4 °C. It has also been taken into account that temperature shows strong inertia (see Chen et al. 2011). For this reason, the catastrophe threshold has been rounded up to 5 °C[18] above the pre-industrial temperature C_P. That threshold is also justified by the uncertainty surrounding the volatility of the temperature process. Indeed, Weitzman (2007) argued that the probability of temperatures exceeding the interval estimated by IPCC (2013) is not negligible, so catastrophe is a definite possibility. With respect to the percentage of GDP invested in mitigation and in adaptation, numerical results show that mitigation expenditures should be higher than adaptation expenditures (Table 2). This might be far different from what actually happens.[19] The parameters for the Monte Carlo simulation are presented in Table 1.

The drift a of the temperature process is positive, given global warming, but very small. The rate chosen is $a = 0.035$. Despite the fact that global temperature increased by 0.8–1.0$°C$ during the last century, the future is very uncertain given current carbon emissions and polluting trends. The parameter chosen thus corresponds to the lower, conservative, limit of the possible temperature increase in the next 100 years (Lenton et al. 2008), and also to

[18] While IPCC (2013) considers that such a temperature increase may occur before the end of the twenty-first century, scientific evidence and opinions about climate sensitivity and temperature processes are very heterogeneous.

[19] Indeed, due to the uncertainty involved, countries might prefer to invest in adaptation rather than in mitigation. Said differently, mitigation techniques, at least at the moment, are much slower than adaptation techniques for reaching the desired goals. Governments are also concerned with the problem of *free riding*, which accompanies many mitigation efforts. This point must be stressed. The *local* characterization of many adaptation projects reduces the issue of free riding, and it represents an additional reason why adaptation is preferred.

Table 1 Model parameters

Parameter	Description	Value	Sensitivity
C_P	Pre-industrial global average temperature level	14.0 °C	–
C_0	2011 global average temperature level	14.8 °C	–
k^*	Optimal mitigation investment fraction	–	(0–10 %)
L^*	Optimal investment threshold	–	(14–22)
L_1	Catastrophe threshold	19	–
μ	Initial drift of the GDP process	3.0 %	–
σ	Volatility of the GDP process	10 %	–
a	Drift of the temperature process	0.035 °C	–
η	Natural trend of global average temperature	−0.1 %	–
α	Parameter modelling the impact of mitigation efforts on the temperature drift	1	–
β	Volatility of the temperature process	–	(0–2 °C)
r	Discount rate	1.5 %	(0.0–5.0 %)
δ	Depreciation rate of the mitigation effort	0	–
ρ	Impact of the temperature gap on the disposable GDP	0.29 %	–
θ	Parameter modelling the magnitude of the delay	$1 Trillion	–
Δt	Timesteps of the processes	1 (year)	–
D	*Parisian* Window	50 (years)	(0; 50)
V_0	GDP in 2011	$69.993 (2011 Trillions)	–
$DGDP_0$	Disposable GDP in 2011	$69.832 (2011 Trillions)	–
T	Horizon	500 years	–

the medium sensitivity as found by Bahn et al. (2012). The volatility of the temperature process was chosen to take different possible values, from $\beta = 0\,°\mathrm{C}$ to $\beta = 2\,°\mathrm{C}$, in order to better reflect the uncertainty that still surrounds climate and temperature models used for forecasting and to better show the influence of such parameter on our results. The outcome of the maximization problem is sensitive to the volatility. Given the variability that surrounds the global average temperature and its anomalies, it did not seem reasonable to limit our analysis to only one fixed standard deviation parameter. Because uncertainty is a core element in climate change, it is necessary to include it in the in the decision maker's optimal choice. Considering that past temperatures and their variability are strongly dependent on frequency, place, and tool of observation (IPCC 2014; Hansen et al. 2010), a volatility parameter ranging from $\beta = 0\,°\mathrm{C}$ to $\beta = 2\,°\mathrm{C}$ seems reasonable. While standard deviations of $\beta = 0\,°\mathrm{C}$ and $\beta = 2\,°\mathrm{C}$ are unlikely to be observed, it is very useful to test the model's behaviour at the extremes.

The starting GDP value, V_0, was chosen to be \$69.993 trillion US dollars, and the drift μ of the GDP process was set to 3 %. Both these values were chosen based on the latest reports on macroeconomic data (Central Intelligence Agency 2012; The World Bank 2012), which in particular show an average annual drift in 2011 of 2.2 % for developed countries and 4.1 % for developing countries. It is likely that more resources are spent on mitigation and adaptation in developed than in developing countries, but this distinction is not taken into account in our analysis. The volatility σ of the GDP process was chosen to be 10 % (World Bank Historical GDP Data).

The parameter δ models whether the mitigation effort is a constant proportion of disposable GDP ($\delta = 0$), or if it decreases ($\delta > 0$), or increases ($\delta < 0$) over time. It was chosen to be zero to simplify the optimization process. The parameter D represents the Parisian delay, i.e., the time window during which the temperature process has to stay above the threshold L_1 for the catastrophe to happen.[20] The choice of $D = 50$ years as the time window is somewhat arbitrary. It is a very short time in climatic terms, in line with the notion of a *tipping point* as discussed by Lenton et al. (2008).[21]

4.1 Optimal temperature threshold and investment rate

Given the above parameters, the optimal mitigation investment threshold L^* is determined by allowing candidate values to vary between 14 and 22°C in successive calculations of the objective function in 14. Numerical simulations that utilize a grid search methodology compute the expected discounted sum of future global disposable GDPs associated with waiting for temperature to reach a level L^* before investing a fraction k^* in mitigation technologies. The optimal fraction k^* of disposable GDP that Governments can invest in mitigation is chosen by allowing k to vary between 0 and 10 %. In fact, higher investment percentages are simply not realistic and the 10% limit never constrains the optimal value.[22] The optimal levels L^* and k^* for various values of β are shown in Table 2. This means that a solution (k^*, L^*) exists for each set of input parameters used. As discussed at the end of Sect. 3.2, a solution might not exist for other parameter combinations, meaning in that case that the optimum decision is never to implement the mitigation policy. As to unicity the issue of multiple solution did not arise in any of the simulations.[23]

The optimal mitigation investment threshold varies between a minimum of 14 °C and a maximum of 15.75 °C depending on the assumed volatility of the temperature process. This means that the threshold date is already behind us (for low assumed temperature volatilities) or is not far in the future (for higher volatilities). How far in the future? We examine this question below (Table 2).

This result has implications that are manifest in terms other than the timing of the mitigation investment decision. If we look at the optimal fraction of disposable GDP that needs to be invested, year after year, once the decision to mitigate is taken, we find that k^* lies between 1

20 The concept of Parisian delay has been borrowed from Parisian options, i.e., financial options whose exercise is triggered by the length of the excursion of a price process above a threshold (Chesney et al. 1997 as well as Chesney and Gauthier 2006).

21 While *tipping elements* may be very heterogeneous and not yet entirely known, for the sake of simplicity they are usually considered to trigger the same effects at the same time. Again as in Lenton et al. (2008), only *tipping elements* caused by human activities are taken into consideration.

22 Indeed, although we do not model this phenomenon, Bahn et al. (2012) found that highly effective adaptation measures can hinder investments in mitigation in the medium to long term.

23 In the maximization process, k^* and L^* are jointly and endogenously determined by relying on a grid search method. In case of multiple local maxima for L, we take the supremum, so that our solution is always unique.

Table 2 Simulation results $r = 1.5\,\%$, $D = 50$ years

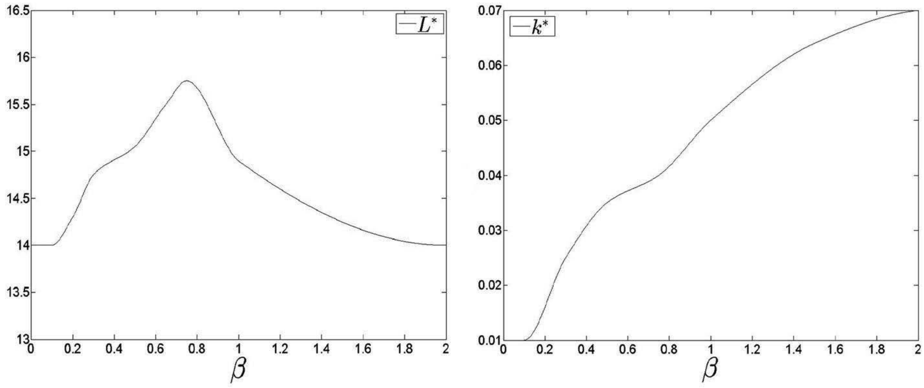
β (°C)	L^* (°C)	k^* (%)
0	14.0	1
0.1	14.0	1
0.3	14.75	2.5
0.5	15.0	3
0.75	15.75	4
1	14.9	5
1.5	14.3	6.8
2	14.0	7

and 7 % depending on the volatility of the temperature process. Furthermore, for small values of the volatility, there is a positive correlation between the optimal temperature threshold and the optimal investment fraction: the higher the temperature threshold, the longer the mitigation decision is postponed (optimally), and the higher the optimal investment effort. This makes intuitive sense since the impact on the temperature drift will need to be stronger if temperature is allowed to come closer to the catastrophe threshold L_1 before any intervention. This positive correlation between the optimal investment threshold L^* and the optimal amount to be invested in mitigation k^* is observed at all reasonable levels of the temperature volatility. It breaks down at high volatilities for reasons discussed below.

As implied by the results stated so far, the volatility of the temperature process is a crucial parameter. This is why the optimal levels of L^* and k^* are presented for different possible values of β in Table 2.

When expressing the optimal investment threshold L^* as a function of the volatility β of the temperature process, it is interesting to note how strong the relationship is between the two. In fact, the level L^* is driven by the uncertainty surrounding future temperature levels in the following way: when uncertainty is fairly low, i.e., $\beta \leq 0.5\,°\mathrm{C}$, it is easier to foresee an increase in temperature levels in the future, given that the drift a of the process is positive, i.e., $a = 0.035$ per unit of time. In this case, it is optimal to invest as soon as possible. Conversely, when the volatility of the temperature process is fairly high, i.e., $\beta \geq 0.50\,°\mathrm{C}$, there is greater uncertainty about future temperature levels. In fact, in this situation, the probability of lower temperatures is higher than before. For $\beta \leq 0.75\,°\mathrm{C}$, results are in line with traditional real options theory: the investment boundary L^* should be an increasing function of the volatility. However, when volatility is very high, this no longer holds true. In this case, the presence of a possible catastrophe provides incentives to mitigate greenhouse gas emissions sooner. Indeed, L^* grows only when β goes from 0 to $0.75\,°C$.

This *non-monotonic* relationship between optimal mitigation temperature and uncertainty is also different from what has been reported in some of the literature on tipping points (Keller et al. 2004; Brozović and Schlenker 2011). The relationship is reversed in our model. In fact, in the presence of a stochastic temperature process, given the ultimate impact of a catastrophe, i.e., a permanent collapse of the global GDP and a delay between the breach of the tipping point L_1 and the occurrence of the catastrophe, the decision maker faces a trade-off: strong uncertainty makes him cautious when risk increases. In this case, his objective is to invest sooner, since the gain from waiting is not worth the additional expected cost linked to the catastrophe. On the contrary, an increase in the uncertainty level makes the gain from waiting the dominating strategy at lower risk levels.



(a) **(b)**

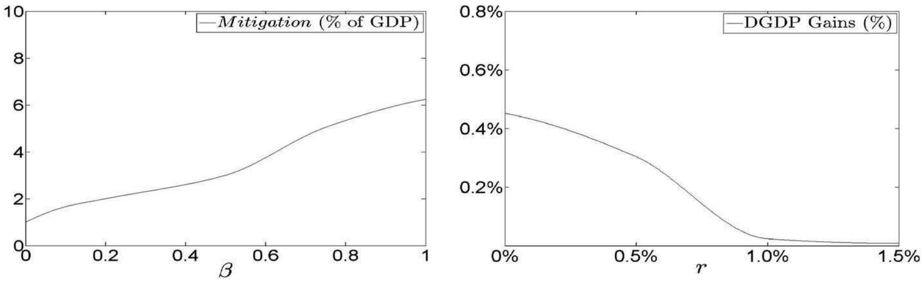
Fig. 2 Optimal investment threshold L^* (Fig. 2a) and optimal mitigation investment fraction k^* (Fig. 2b), plotted against volatility β, for $D = 50$

In the standard real options theory, the optimal boundary is a monotonic function of the risk. However, in some specific cases, in particular for double barrier American currency calls, this non-monotonic feature might also be observed. With this specific option, if the value of the currency reaches one of the barriers, then the option is lost. The loss of the option plays the role of the catastrophe in the framework of this model. With such a barrier option, an increase in volatility generates a higher exercise boundary when volatility levels are small. However, with higher levels of the volatility, opposite effect appears: the exercise boundary decreases when volatility increases. There is a trade-off between the potential benefit that a volatility increase might generate, i.e., higher profit, and potential risks, i.e., higher probability of losing the option. For small levels of volatility, the first effect dominates; for higher levels, the second one is stronger.

Concerning the effects on the optimal mitigation investment fraction k^*, this leads to increases in β because the more volatile the temperature process is, the stronger the financial effort needs to be in order to bring the temperature process back to acceptable levels. For $\beta > 0.75\,°\mathrm{C}$, the optimal investment fraction k^* keeps increasing but at a slower pace. This behaviour is caused by the lower optimal investment threshold, which allows for smaller increases in k^*. Figure 2a, b illustrate the relationships just mentioned.
The right part of Fig. 2a can be justified by looking at the behaviour of the expected catastrophe date as a function of risk, pictured in Fig. 6b. When volatility grows, the expected date decreases, making a high -and prompt- monetary investment necessary.

An important relationship to look at is the one occurring at T_L, between the optimal amount to be invested in mitigation k^* and the volatility β. In our model, β is one of the parameters determining the impacts of climate change on the expected discounted sum of future GDP. Optimal mitigation efforts are an increasing function of the temperature volatility. Even with the smallest volatilities, mitigation efforts are at least equal to 1 % of the GDP as pictured in Fig. 3a. This percentage is much higher than currently observed levels. It is also much higher than the exogenous adaptation effort estimated to be 0.29 % in this model. This justifies prompt action in mitigation, in line with the latest international reports on climate change (IPCC 2014).

Figure 3b shows the percentage gains obtained from undertaking mitigation efforts for $\beta = 0.75\,°\mathrm{C}$ in terms of disposable GDP and for different levels of the interest rate r. The



(a) **(b)**

Fig. 3 **a** Mitigation Efforts, as fraction of GDP, plotted against volatility β. **b** Cumulative DGDP gains (%) of mitigation versus adaptation only, as function of r and for $\beta = 0.75\,°\mathrm{C}$

graph is obtained by comparing the expected sums of discounted disposable GDP with and without mitigation, i.e., when mitigation efforts correspond to the optimal k^* or to zero. These gains are a decreasing function of the discount rate r. Indeed, when this rate is high, costs generated by a future catastrophe, when discounted, might appear almost negligible. When the discount rate is small, then a strong mitigation effort is required in order to decrease environmental risks. In this case, interests of future generations are taken into account.

The relationship between mitigation and adaptation, and their impacts on the temperature and on the disposable GDP process, is illustrated in Fig. 4. The upper part of the plot shows the temperature process and its two different dynamics, without mitigation (green line; $k = 0\,\%$) and with optimal mitigation (black line; $k^* = 4\,\%$), for $\beta = 0.75\,°\mathrm{C}$. The lower part of the plot shows the same relationship but expressed in terms of its impacts on the disposable GDP, for $r = 0$. In this case, it is clear that investing in mitigation at time T_L has caused the drift of the temperature process to shift down at time $T_L + \Delta T(k, L)$, from $a(0) = a$ to $a(0.04) < a$. As a consequence, a catastrophe before $T = 500$ years has been avoided. This allows for a higher discounted sum of future GDPs, given by the blue area below the curve. Indeed, despite a lower disposable GDP as of T_L, due to the fact that a fraction $k^* > 0$ has to be invested in mitigation each year, there are significant positive GDP flows instead of zero (once the catastrophe has occurred) at later stages. These significant gains accrue to future generations only. In the particular case pictured, this makes the strategy of investing in mitigation the optimal k^*, i.e., $k^* = 4\,\%$ for $\beta = 0.75\,°\mathrm{C}$, preferred to the strategy of doing nothing, i.e. no investment in mitigation.

As argued in Sect. 1, the possibility of a climate catastrophe occurring and the impact of mitigation decisions on its probability are arguably the most important questions facing decision makers. For the stochastic processes used in this paper, the catastrophe is certain to happen given a distant enough time horizon. However, it is possible to compute its expected date if nothing is done and its expected date under the optimal policy as well as the sensitivity of these expected dates to parameters. Although the expected date of a catastrophe occurring is reduced by the optimal policy, we find that the possibility of its occurrence within the next 500 years is far from remote for small levels of risk. Figure 7a illustrates this idea. The probability of the catastrophe occurring within the next 500 years is between 5 % and 100 % depending on β. Given the proximity of that occurrence, we investigate its various determinants further below.

To sum up the main results, depending on the volatility of the temperature process, the optimal mitigation investment threshold varies between a minimum of 14 °C and a maximum of 15.75 °C, and the optimum proportion of GDP invested in mitigation lies between 1 and

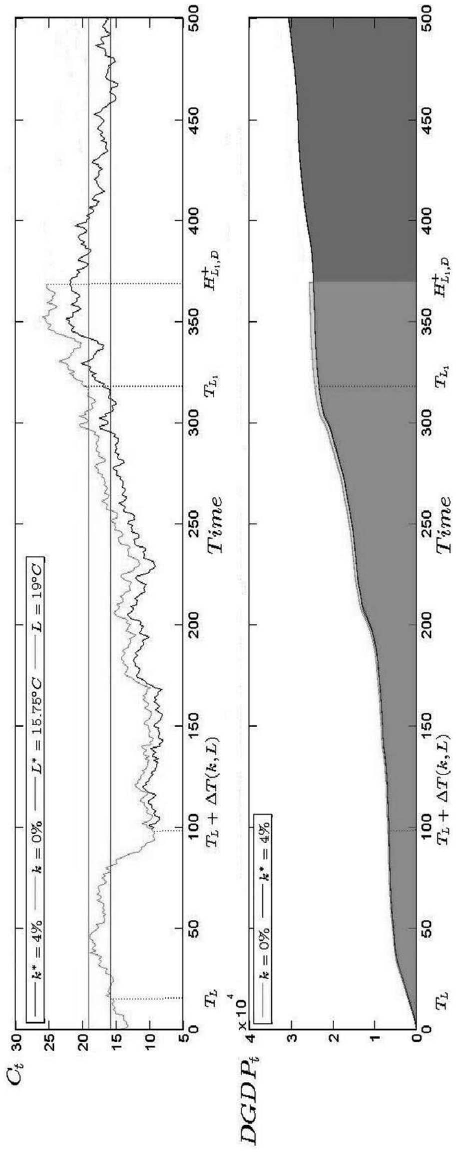


Fig. 4 Impacts of mitigation versus adaptation only on the temperature process and on the sum of discounted future disposable GDPs, for $\beta = 0.75\,°\mathrm{C}$

7 %. This proportion is much higher than currently observed levels. It is also much higher than the exogenous adaptation effort estimated to be 0.29 % in this model. The gains from adopting an optimal mitigation policy are significant; they accrue to future generations only while current generations incur costs.

There is positive correlation at realistic values of temperature volatility between the optimal investment threshold and the optimal amount to be invested in mitigation; this correlation breaks down at unrealistically high volatilities. In contrast, the relationship between the optimal mitigation investment and temperature volatility is positive: the more volatile the temperature process is, the stronger the financial effort needs to be in order to bring the temperature process back to acceptable levels.

Finally, perhaps more concretely, the probability of the catastrophe occurring within the next 500 years lies between 5 % and 100 % depending on β. Indeed the volatility of the temperature process is probably the most important exogenous factor affecting the results. In the next section we conduct sensitivity analyses to changes in volatility and other key model parameters.

4.2 Sensitivity analysis

Monte Carlo simulations were run using different values for the exogenous variables β, D, and r to check their impact on the optimal choice that must be determined by the global decision maker in the presence of global warming. Parameters that need to be considered are the optimal temperature threshold L^* that triggers the optimal investment in mitigation as well as k^*, i.e., the fraction of disposable GDP to be invested in mitigation to achieve the maximum expected discounted sum of future GDPs.

Figure 5 illustrates the maximum values of L^* and k^*, plotted against different values of the Parisian window D, for $\beta = 0.75\,°\mathrm{C}$. It is interesting to notice that, while the optimal threshold L^* remains constant when the excursion length D increases, the optimal fraction of GDP to be invested in mitigation k^* shows a decreasing behaviour. In other words, arguments based on the assumption that long time windows allow for a greater time delay in the social planner's decision-making process do not seem to be justified: a longer window does not imply a greater delay before investing in mitigation, but only a lower fraction k^* of GDP to be devoted to mitigation measures.

As we see in Fig. 6a, for small values of the risk, the expected date T_L, i.e. the moment when the threshold L is reached, increases when the volatility β increases. It should not to be forgotten that when $\beta \leq 0.75\,°\mathrm{C}$, the optimal temperature L^*, which triggers the investment in mitigation k^*, increases when β increases. Therefore, it becomes more likely that the barrier L^* is also crossed at later stages. When β is higher than $0.75\,°\mathrm{C}$, L^* starts decreasing again, thus the expected date T_L decreases as well, since crossing the optimal investment threshold might happen sooner.

Figure 6b shows the expected date of the environmental catastrophe when it is smaller than T, i.e., when it happens within the chosen time horizon. The expected date decreases when the volatility β increases. Without uncertainty (i.e., $\beta = 0$), this expected date in the business-as-usual scenario is 175 years. As risk increases, a catastrophe might happen sooner and the expectation decreases.[24]

Figure 7a shows the probability that both events T_L, the moment when the threshold L that triggers the investment in mitigation k, and the catastrophe, happen within $T = 500$ years, plotted against the volatility β. The probability of both $T_L \leq T$ and $H^+_{L_1,D} \leq T$

24 As we are not considering trajectories for which $H^+_{L_1,D} > T$.

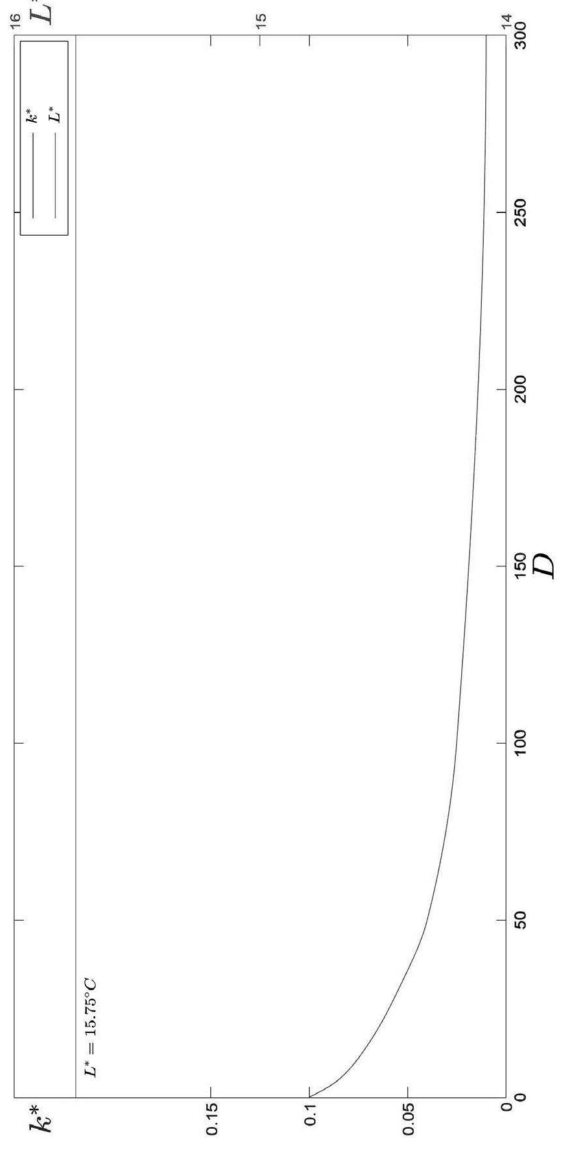


Fig. 5 Optimal mitigation investment fraction k^* and optimal investment threshold L^*, plotted against different values of the Parisian Window D, for volatility $\beta = 0.75\,°\mathrm{C}$

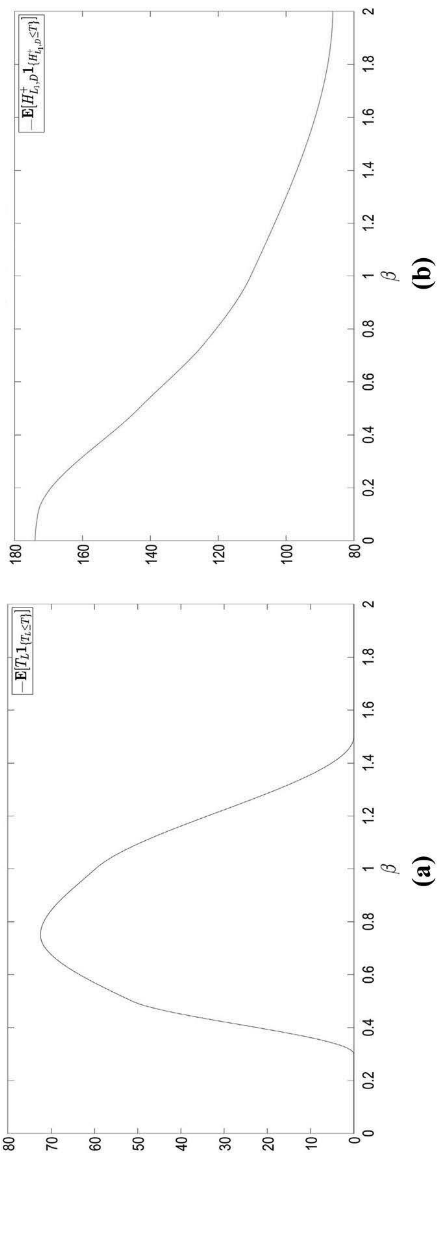


Fig. 6 **a** Expected date T_L plotted against volatility β, for $D = 50$. **b** Expected date of the environmental catastrophe $H^+_{L_1,D}$ plotted against volatility β for $D = 50$

decreases with an increase in the volatility parameter β. This relationship is expected: when the volatility of the temperature process increases, the probability that such a process moves away from the barrier L_1 and a) never touches it or b) decreases and goes back below it increases as well. In addition, as seen in Fig. 7a, the probability of the event $H^+_{L_1,D} \leq T$ is always smaller than the probability of the event $T_L \leq T$, because the optimal threshold L^* is lower than the tipping point L_1.

Figure 7b shows the probability of the event $T_L \leq T$, and the probability of the catastrophic event $H^+_{L_1,D} \leq T$, plotted against different possible values of the Parisian window D. As expected, the probability of the event $T_L \leq T$ remains constant for a given volatility when the Parisian window D increases, since this has no effect on where and when the temperature process crosses the barrier L^* and thus triggers the optimal investment in mitigation k^* from the global decision maker. Conversely, the probability of event $H^+_{L_1,D} \leq T$ decreases, going from almost 50 % to about 6.9 %.

Figure 7c illustrates the probability $P(H^+_{L_1,D} - T_{L_1} = D)$, i.e., the probability that a catastrophe will occur D units of time after the first time the temperature process passes the tipping point L_1. As expected, the longer the Parisian window D, the lower the probability $P(H^+_{L_1,D} - T_{L_1} = D)$. However, it can be clearly seen that such a probability remains fairly high, i.e., above 80 %, even for very long Parisian windows D. The presence of a large time window does not imply that a catastrophe should be neglected.

4.3 A deterministic temperature process

Finally, as mentioned in Sect. 3, various values of α could be used; α defines the proportion of disposable GDP that needs to be invested in order to eliminate anthropogenic elements in the determination of the temperature drift. Setting $\alpha = 0.1$ instead of $\alpha = 1$, and assuming a deterministic temperature process, we obtain the results shown in Fig. 8 by relying on the analytical approach explained in “Appendix 3”. It shows that the optimal drift $a(k)$ then becomes negative provided the discount rate is small enough i.e. provided the interests of future generations are given enough weight. In this case, it is optimal to avoid the catastrophe. With a higher discount rate, business as usual in terms of emissions leads to a global catastrophe before the horizon T equals 500 years. Long-term catastrophes are almost negligible today, when discounted at standard levels of interest rates. Unfortunately, only a small discount rate will generate incentives to curb CO_2 emissions and therefore decrease the drift in temperature.

4.4 Summary of sensitivities

Sensitivity analyses allow us to shed light on important model results. First of all, as pictured in Fig. 5, the length of the Parisian window D does not have a significant impact on the optimal investment threshold L^*. However, it has an impact on the optimal fraction k^* of GDP to be invested in mitigation, which decreases when the time window increases. This is indicative of the fact that larger investment is needed when the Parisian window is small, because the temperature process needs to be brought down to acceptable levels more quickly. Higher values for k^* reduce the drift $a(k)$ of the temperature process.

The expected date $E[T_L]$, which indicates the expectation of the first passage time of the temperature process above the optimal investment threshold L^*, initially increases as volatility β increases. As soon as volatility crosses a critical level, the expected passage time starts decreasing again (Fig. 6a). This behaviour closely resembles the behaviour of the optimal temperature threshold L^* when expressed as a function of the volatility β, as can

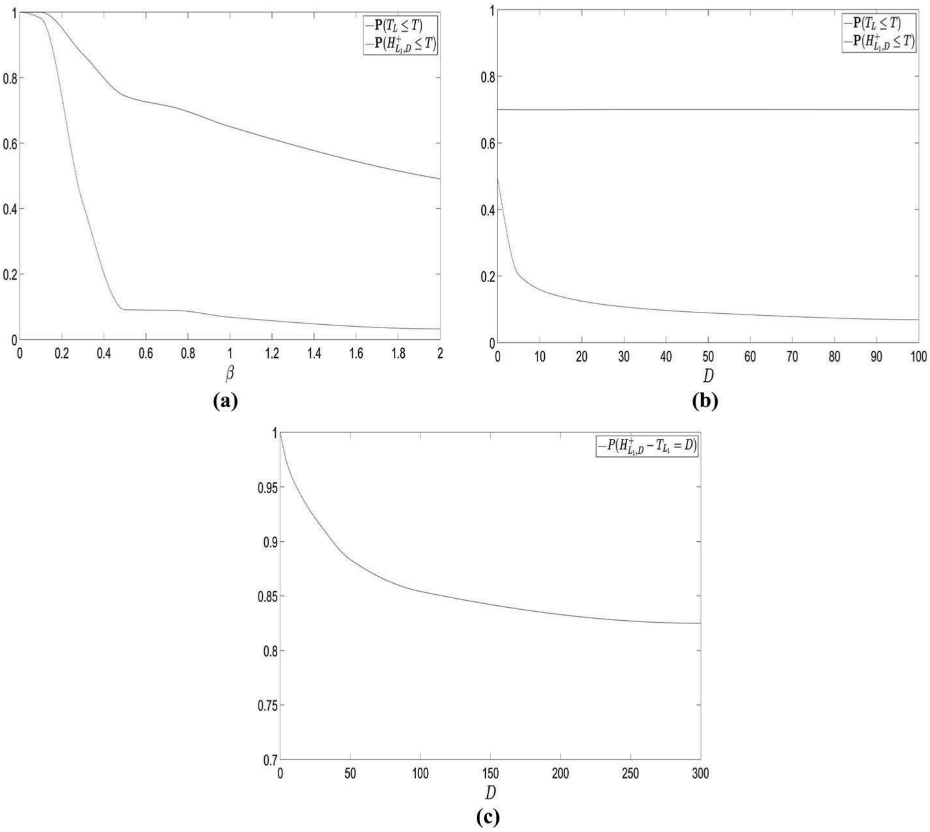


Fig. 7 **a** Probability of $T_L \le T$ and probability of $H^+_{L1,D} \le T$, plotted against volatility β, for $D = 50$. **b** Probability of $T_L \le T$ and probability of $H^+_{L1,D} \le T$, plotted against different lengths of the Parisian Window D, for $\beta = 0.75\,°\mathrm{C}$. **c** Probability that a catastrophe occurs D units of time after T_{L_1}, plotted against different lengths of the Parisian Window D, for $\beta = 0.75\,°\mathrm{C}$

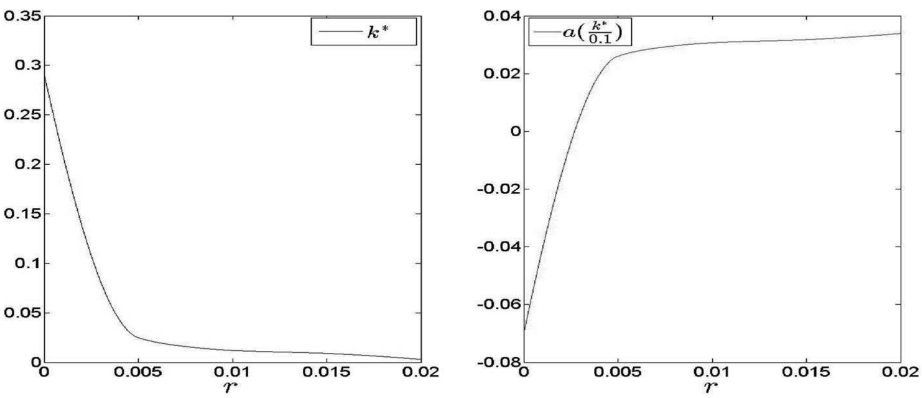


Fig. 8 Optimal mitigation investment fraction k^* and temperature Drift $a(k^*)$, plotted against interest rate r, for volatility $\beta = 0\,°\mathrm{C}$ and $\alpha = 0.1$

be seen in Fig. 2a. The reason can be found in the fact that when the optimal investment threshold L^* decreases, the temperature process might cross it more easily. Conversely, the expected dates $E[T_{L_1}]$ and $E[H^+_{L_1,D}]$, which indicate respectively the expected first passage time of the temperature process above the *tipping point* L_1 and the expected date of the catastrophe, are both monotonically decreasing functions of volatility β. In fact, as volatility β increases, both the event T_{L_1} and $H^+_{L_1,D}$ could happen sooner (see Fig. 6b).

The probabilities of T_{L_1} and $H^+_{L_1,D}$ taking place before $T = 500$ years decrease as volatility increases. In fact, as volatility increases, temperatures are likely to move away from the *tipping point* L_1, either never reaching it, or going back below it once crossed, thus making the occurrence of events T_{L_1} and $H^+_{L_1,D}$ less probable before the horizon T.

5 Conclusion

Most IAM assume gradual environmental degradation and ignore the possibility of abrupt events. This paper has emphasized the possibility of a climate catastrophe and the nature of the policy decisions implied. The literature proposes many ways to model catastrophes, including some from which society recovers. We have chosen to envisage a 'final' catastrophe, the aftermath or which is impossible to predict or assess. Such a catastrophe may happen if earth temperature reaches and exceeds some threshold. While many researchers have assumed the catastrophe threshold to be uncertain, which implies that a catastrophe will not occur if temperature stays below any previously reached level that did not trigger a catastrophe, we assume that the threshold is known but that the catastrophe happens only if the threshold is exceeded for a long enough period (of 50 year in our application). Our model reflects the implied policy options: it may not be optimal but it is possible to delay mitigation decisions until global temperature is dangerously high; the catastrophe might be avoided even if the threshold that triggers it has been reached.

Besides climate modelling, this paper attempts to reflect the institutional and technological context under which policy decisions are arrived at. As illustrated by the results of the COP 21 in Paris in December 2015, policies often take the form of a "one shot" decision that is irreversible and requires dedicating resources. The timing of the policy and the magnitude of the engagement must be chosen optimally. This consideration has motivated the adoption of a real option formulation as that methodology is particularly well adapted to such optimal stopping problems.

Real options models have mostly emphasized the timing of decisions. Our model innovates in that the choice of the optimum date for implementing the mitigation decision is combined with the choice of the optimal mitigation effort. For a discount rate of 1.5 % and with a temperature volatility of 0.3 °C, Monte Carlo simulations show that governments should invest 2.5 % of disposable GDPs in mitigation when the temperature process hits 14.75 °C in order for the world to achieve the maximum possible expected discounted sum of future disposable GDP. Unfortunately, global temperature level has already reached this threshold. If the volatility of the temperature process is 0.75 °C, the optimal investment boundary increases to 15.75 °C, implying that there may still be some time left to invest in mitigation, although a larger investment must then be devoted to it.

In any case investing in mitigation is urgent, and adaptation cannot be considered a substitute. Indeed, the optimum proportion of GDP that should be invested in mitigation lies between 1 and 7 %. This proportion is much higher than currently observed levels. It is also much higher than the exogenous adaptation effort estimated to be 0.29 % in this model.

The relationship between the optimal temperature threshold and temperature volatility is not monotonic; the optimal threshold diminishes when the assumed volatility increases beyond about .75 °C. This is because two opposing effects determine the optimal temperature threshold. The first effect is that one may regret an early mitigation decision if the temperature is not to increase as quickly as forecast while the expenditure in GDP is irreversibly committed. Such a source of regret is less likely when volatility is low because forecasts are then more reliable, so that the mitigation decision may be taken early if volatility is low and should be postponed if volatility is high. The second effect is that, given any current temperature, the probability for temperature to reach the level that may trigger the catastrophe within a given period is higher, the higher the volatility. Precaution then calls for earlier mitigation, the higher the volatility. In our simulations, the first effect dominates at low volatilities; the second effect dominates at high volatilities. In other words, the decision maker is confronted with a trade-off between the irreversibility associated with the investment in mitigation and the irreversibility associated with the climate catastrophe in the absence of investments in mitigation. In this setting, the optimization process implies for the policy maker that the date to start mitigation has already been exceeded both if the actual volatility is lower than about 0.3 °C and if it is higher than 1 °C but not for intermediate volatility levels.

The impact of volatility on the optimal investment rate turns out to be monotonic unlike the impact on the temperature threshold. This is because a third element enters the investment decision. When volatility is high, the effort necessary to keep temperature away from dangerous levels is higher. Accordingly, even if the expected optimal date of implementation decreases when volatility rises at high volatility, which gives mitigation more time to be effective, the risk of catastrophe associated with high temperature fluctuations calls for a commitment of higher magnitude than when volatility is low.

All in all, a model is only a model and our's is not immune to Pindyck's (Pindyck 2015) criticism that models can be packed with the assumptions necessary to generate the conclusions desired by their authors. Nevertheless, if one believes that a climate catastrophe is a possibility to take into consideration and that a catastrophe is not just another mishap, the assumptions adopted in this paper are moderate, transparent, and justified by the observation of reality. It is also comforting that the message to be drawn from our paper is not very different from what other researches involving climate catastrophes conclude. Mitigation cannot be dispensed with. Mitigation is urgent. Adaptation is not a substitute for mitigation. The effort that needs to be extended will be higher, the longer mitigation is postponed.

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Appendix 1: Some mathematical tools

For $x = C_0$, we have $C_t = x + at + \beta W_t$, or $C_t = x + \beta Z_t$, with Z_t a *drifted* $\mathbb{Q}$−Brownian motion, i.e., $(Z_t = \gamma t + W_t, t \geq 0)$, with $\gamma = \left(\frac{a}{\beta}\right)$.

Let us now define the following functions in terms of T (Chesney et al. 1997):

$$T_{L_1}(C) = \inf\{t \geq 0 : C_t \geq L_1\} \tag{17}$$

$$g_t^{L_1}(C) = \sup\{s \in [0, t] : C_t \geq L_1\} \tag{18}$$

$$H^+_{L_1,D}(C) = \inf\{t \geq 0 : \left(t - g_t^{L_1}(C)\right) \geq D \quad \text{and} \quad C_t \geq L_1\} \tag{19}$$

or in terms of Z

$$T_{L_1}(Z) = \inf\{t \geq 0 : Z_t \geq l_1\} \tag{20}$$

$$g_t^{l_1}(Z) = \sup\{s \in [0, t] : Z_t \geq l_1\} \tag{21}$$

$$H^+_{l_1,D}(Z) = \inf\{t \geq 0 : \left(t - g_t^{l_1}(Z)\right) \geq D \quad \text{and} \quad Z_t \geq l_1\} \tag{22}$$

with

$$l_1 = \frac{L_1 - C_0}{\beta}.$$

They are, respectively, the first instant the temperature process hits the given level L_1, the last instant before t when this process was at a given level L_1, and the Parisian time, i.e., the first instant when the process spends consecutively D units of time over the level L_1. Notice that $g_t^h(C)$ is not a stopping time. When this random time happens, there is no way to know immediately that it has just happened. We will note $H^+_{L_1,D}$ for $H^+_{L_1,D}(C)$.

The mathematical tools useful in this context are the following:

1. The random variables $H^+_{l_1,D}$ and $Z_{H^+_{l_1,D}}$ are independent
2. The law of $Z_{H^+_{l_1,D}}$ is known

$$P(Z_{H^+_{l_1,D}} \in dy) = \frac{dy}{D} \mathbb{I}_{y>L}(y - l_1) \exp\left(-\left(\frac{(y - l_1)^2}{2D}\right)\right) \tag{23}$$

with $y = (Z_{H^+_{l_1,D}} - Z_{T_{L_1}})$

3. The Laplace transform of $H^+_{L_1,D}$ is given by Chesney et al. (1997)

$$E\left(\exp\left(-\frac{\lambda^2}{2} H^+_{L_1,D}\right)\right) = \frac{\exp(l_1\lambda)}{\Phi(\lambda\sqrt{D})} \tag{24}$$

where the function Φ is known

$$\Phi(y) = \int_0^{+\infty} z \exp\left(zy - \frac{z^2}{2}\right) dz = 1 + \sqrt{2\pi}\, y e^{\frac{y^2}{2}} \mathcal{N}(y) \tag{25}$$

and

$$\mathcal{N}(y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{y} e^{-\frac{x^2}{2}} dx. \tag{26}$$

Appendix 2: The boundedness argument

By relying on Section 2.1 of the textbook entitled "Optimal Stopping and Free Boundary Problems" by Peskir and Shiryaev (2006), if the following condition is satisfied, the problem is mathematically well-defined:

$$\begin{aligned} X \equiv E_{\mathbb{P}} \sup_{0 \leq t \leq T} \Bigg| &\int_{t \wedge T}^{t+\Delta T(k,L) \wedge H^+_{L_1,D} \wedge T} DGDP_u\,(1-k)\,e^{-ru} du \\ &+ \int_{t+\Delta T(k,L) \wedge H^+_{L_1,D} \wedge T}^{H^+_{L_1,D} \wedge T} \widehat{DGDP}_u\,(1-k)\,e^{-ru} du \\ &- \int_{t \wedge T}^{T} DGDP_u e^{-ru} du \Bigg| < +\infty \end{aligned} \tag{27}$$

The measure $\mathbb{P}$ used in the expectation is related to is the so-called historical probability, that is to say the probability which reflects historical data.
The expectation can be rewritten as follows:

$$\begin{aligned} X = E_{\mathbb{P}} \sup_{0 \le t \le T} \Bigg| &- k \int_{t \wedge T}^{t+\Delta T(k,L) \wedge H^{+}_{L_1,D} \wedge T} DGDP_u e^{-ru} du \\ &- \int_{t+\Delta T(k,L) \wedge H^{+}_{L_1,D} \wedge T}^{T} DGDP_u e^{-ru} du \\ &+ (1-k) \int_{t+\Delta T(k,L) \wedge H^{+}_{L_1,D} \wedge T}^{H^{+}_{L_1,D} \wedge T} \widehat{DGDP}_u e^{-ru} du \Bigg| \end{aligned} \tag{28}$$

The functions in the three integrals are positive and k is smaller than 1. Therefore:

$$X \le E_{\mathbb{P}} \Bigg[\max \Bigg((k+1) \int_0^T DGDP_u e^{-ru} du, \quad (1-k) \int_0^T \widehat{DGDP}_u e^{-ru} du \Bigg) \Bigg] \tag{29}$$

Given the definition of DGDP in Eq. (3) and of $\widehat{DGDP}$ in Eq. (13):

$$X \le E_{\mathbb{P}} \Bigg[\max \Bigg((k+1) \int_0^T V_u e^{-ru} du, \quad (1-k) \int_0^T V_u e^{-ru} du \Bigg) \Bigg] \tag{30}$$

i.e.,

$$X \le (k+1) E_{\mathbb{P}} \int_0^T V_u e^{-ru} du \tag{31}$$

Given the definition of the process V in Eq. (2), and by relying on Ito's lemma:

$$X \le (k+1) E_{\mathbb{P}} \left(\int_0^T e^{-(r-\mu)u} \cdot e^{-\frac{\sigma^2}{2}u + \sigma B_u} du \right) \tag{32}$$

The process $\{e^{-\frac{\sigma^2}{2}u+\sigma B_u},\ u \ge 0\}$ is a $\mathbb{P}-$martingale; therefore by relying on Fubini's theorem:

$$X \ \le \ (k+1) \int_0^T e^{-(r-\mu)u} du \ \le \ \frac{(k+1)}{r-\mu} \left(1 - e^{-(r-\mu)T}\right) \ < \ +\infty \quad \square \tag{33}$$

Appendix 3: The particular case of a deterministic temperature process

Recall our objective function:

$$\sup_{k,L} f(k,L) \Leftrightarrow \sup_{k,L} \left[\mathbf{1}_{L<L_1} \cdot g_1(k,L) + \mathbf{1}_{L \ge L_1} \cdot g_2(k,L) \right] \tag{34}$$

with

$$g_1(k, L) = E_{\mathbb{P}}\left[\underbrace{\int_0^{T_L \wedge T} DGDP_u e^{-ru} du}_{I_1 a} + \underbrace{\int_{T_L \wedge T}^{T_L + \Delta T(k,L) \wedge H^+_{L_1,D} \wedge T} DGDP_u \left(1 - ke^{-\delta u}\right) e^{-ru} du}_{I_1 b} + \underbrace{\int_{T_L + \Delta T(k,L) \wedge H^+_{L_1,D} \wedge T}^{H^+_{L_1,D} \wedge T} \widehat{DGDP}_u \left(1 - ke^{-\delta u}\right) e^{-ru} du}_{I_1 c}\right] \tag{35}$$

and

$$g_2(k, L) = E_{\mathbb{P}}\left[\underbrace{\int_0^{T_L \wedge H^+_{L_1,D} \wedge T} DGDP_u e^{-ru} du}_{I_2 a} + \underbrace{\int_{T_L \wedge H^+_{L_1,D} \wedge T}^{T_L + \Delta T(k,L) \wedge H^+_{L_1,D} \wedge T} DGDP_u \left(1 - ke^{-\delta u}\right) e^{-ru} du}_{I_2 b} + \underbrace{\int_{T_L + \Delta T(k,L) \wedge H^+_{L_1,D} \wedge T}^{H^+_{L_1,D} \wedge T} \widehat{DGDP}_u \left(1 - ke^{-\delta u}\right) e^{-ru} du}_{I_2 c}\right] \tag{36}$$

In the absence of volatility, i.e., when $\beta = 0$, and given our set of parameters, as in Table 1, the integrals in Eqs. (35) and (36) can be solved quasi-analytically.
In order to do so, we will rely on a few facts. First, we know that $min(T_L, T) = T_L$ and that $min(T_L + \Delta T(k, L), T) = T_L + \Delta T(k, L)$. Moreover, since all the integrals are bounded by construction, it is possible to use Fubini's theorem to bring expected values inside the integrals.
Furthermore, we will make use of the fact that, given the dynamics of the temperature process, i.e., $dC_t = adt + \sigma dW_t$, we have $C_t = C_0 + at + \beta W_t$. With respect to the dynamics of the GDP process, i.e., $dV_t = \mu V_t dt + \sigma V_t dB_t$, we obtain $V_t = V_0 e^{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma B_t}$.

Other useful equalities that will be extensively used are

$$\delta = 0 \tag{37}$$

$$\alpha = 0.1 \tag{38}$$

$$T_L = \frac{L - C_0}{a} \leq \frac{22 - 14.8}{0.035} = 205.7 < T = 500 \quad \text{for} \quad L \in [14, 22] \tag{39}$$

$$a(k) = a - (a - \epsilon)\frac{k}{\alpha} \tag{40}$$

5.1 $L < L_1$

- Integral $I_1 a$

$$
\begin{aligned}
I_1 a &= E_{\mathbb{P}} \int_0^{T_L \wedge T} DGDP_u e^{-ru} du \\
&= E_{\mathbb{P}} \int_0^{T_L} V_0 e^{\left(\mu - \frac{\sigma^2}{2}\right)u + \sigma B_u} e^{-\rho(C_0 + au - C_P)} e^{-ru} du \\
&= \int_0^{T_L} V_0 e^{(\mu - \rho a - r)u} E_{\mathbb{P}}[e^{-\frac{\sigma^2}{2}u + \sigma B_u}] e^{-\rho(C_0 - C_P)} du
\end{aligned}
$$

We know that $\{e^{-\frac{\sigma^2}{2}t + \sigma B_t}, t \geq 0\}$ is a *martingale*, therefore $E[e^{-\frac{\sigma^2}{2}t + \sigma B_t}] = 1$. We can then write

$$
\begin{aligned}
I_1 a &= V_0 e^{-\rho(C_0 - C_P)} \int_0^{T_L} e^{(\mu - \rho a - r)u} du \\
&= V_0 e^{-\rho(C_0 - C_P)} \frac{e^{(\mu - \rho a - r)T_L} - 1}{\mu - \rho a - r}
\end{aligned}
\tag{41}
$$

where T_L is given by (39).

- Integral $I_1 b$

In Integral 1b, we have to distinguish two different cases, based on whether $T_L + \Delta T(k, L)$ is smaller or greater than $H^+_{L_1,D}$.

$$
\begin{aligned}
I_1 b &= E_{\mathbb{P}} \int_{T_L \wedge T}^{T_L + \Delta T(k,L) \wedge H^+_{L_1,D} \wedge T} DGDP_u (1-k) e^{-ru} du \\
&= E_{\mathbb{P}} \int_0^{T - T_L} \left(\int_{T_L}^{T_L + t \wedge H^+_{L_1,D}} V_0 e^{\left(\mu - \frac{\sigma^2}{2}\right)u + \sigma B_u} e^{-\rho(C_0 + au - C_P)} (1-k) e^{-ru} du \right) \\
&\quad \times \mathbb{P}(\Delta T(k, L) \in dt) \\
&= V_0 (1-k) e^{-\rho(C_0 - C_p)} E_{\mathbb{P}} \int_0^{T - T_L} \left(\int_{T_L}^{T_L + t \wedge H^+_{L_1,D}} e^{(\mu - \rho a - r)u} e^{-\frac{\sigma^2}{2}u + \sigma B_u} du \right) \\
&\quad \times \mathbb{P}(\Delta T(k, L) \in dt)
\end{aligned}
\tag{42}
$$

This integral can be rewritten as

$$
\begin{aligned}
I_1 b = V_0 (1-k) e^{-\rho(C_0 - C_p)} &\Bigg\{ E_{\mathbb{P}} \int_0^{\frac{L_1 - L}{a} + D} \left(\int_{T_L}^{T_L + t} e^{(\mu - \rho a - r)u} e^{-\frac{\sigma^2}{2}u + \sigma B_u} du \right) \\
&\mathbb{P}(\Delta T(k, L) \in dt) \\
&+ E_{\mathbb{P}} \int_{\frac{L_1 - L}{a} + D}^{T - T_L} \left(\int_{T_L}^{\frac{L_1 - C_0}{a} + D} e^{(\mu - \rho a - r)u} e^{-\frac{\sigma^2}{2}u + \sigma B_u} du \right) \mathbb{P}(\Delta T(k, L) \in dt) \Bigg\}
\end{aligned}
$$

We can now apply Fubini's theorem to find

$$
\begin{aligned}
I_1 b = & V_0(1-k)e^{-\rho(C_0-C_p)}\left\{ \int_0^{\frac{L_1-L}{a}+D} \left(\int_{T_L}^{T_L+t} e^{(\mu-\rho a-r)u} E_{\mathbb{P}}\left[e^{-\frac{\sigma^2}{2}u+\sigma B_u} \right] du \right) \right. \\
& \mathbb{P}\left(\Delta T(k,L) \in dt\right) \\
& \left. + \int_{\frac{L_1-L}{a}+D}^{T-T_L} \left(\int_{T_L}^{\frac{L_1-C_0}{a}+D} e^{(\mu-\rho a-r)u} E_{\mathbb{P}}\left[e^{-\frac{\sigma^2}{2}u+\sigma B_u} \right] du \right) \mathbb{P}\left(\Delta T(k,L) \in dt\right) \right\} \\
= & V_0(1-k)e^{-\rho(C_0-C_p)}\left\{ \int_0^{\frac{L_1-L}{a}+D} \left(\int_{T_L}^{T_L+t} e^{(\mu-\rho a-r)u} du \right) \mathbb{P}\left(\Delta T(k,L) \in dt\right) \right. \\
& \left. + \int_{\frac{L_1-L}{a}+D}^{T-T_L} \left(\int_{T_L}^{\frac{L_1-C_0}{a}+D} e^{(\mu-\rho a-r)u} du \right) \mathbb{P}\left(\Delta T(k,L) \in dt\right) \right\}
\end{aligned}
$$

where we relied on the fact that $\left\{ e^{-\frac{\sigma^2}{2}t+\sigma B_t}, t \geq 0 \right\}$ is a *martingale*. We then have

$$
\begin{aligned}
I_1 b = & \frac{V_0(1-k)e^{-\rho(C_0-C_p)}}{\mu-\rho a-r}\left\{ \int_0^{\frac{L_1-L}{a}+D} \left(e^{(\mu-\rho a-r)(T_L+t)} - e^{(\mu-\rho a-r)T_L} \right) \right. \\
& \mathbb{P}\left(\Delta T(k,L) \in dt\right) \\
& \left. + \int_{\frac{L_1-L}{a}+D}^{T-T_L} \left(e^{(\mu-\rho a-r)(\frac{L_1-C_0}{a}+D)} - e^{(\mu-\rho a-r)T_L} \right) \mathbb{P}\left(\Delta T(k,L) \in dt\right) \right\} \\
= & \frac{V_0(1-k)e^{-\rho(C_0-C_p)}}{\mu-\rho a-r}\left\{ \int_0^{\frac{L_1-L}{a}+D} e^{(\mu-\rho a-r)(T_L+t)} \mathbb{P}\left(\Delta T(k,L) \in dt\right) \right. \\
& - e^{(\mu-\rho a-r)T_L}\mathbb{P}\left(\Delta T(k,L) \leq \frac{L_1-L}{a}+D \right) \\
& \left. + \left(e^{(\mu-\rho a-r)(\frac{L_1-C_0}{a}+D)} - e^{(\mu-\rho a-r)T_L} \right) \mathbb{P}\left(\frac{L_1-L}{a}+D \leq \Delta T(k,L) \leq T-T_L \right) \right\}
\end{aligned}
$$

where

$$
\begin{aligned}
\mathbb{P}\left(\Delta T(k,L) \in dt\right) &= \frac{\partial \mathbb{P}\left(\Delta T(k,L) \leq t\right)}{\partial t} \\
&= \frac{\partial \mathbb{P}\left(\frac{\theta}{V_{T_L} k e^{-\rho(L-C_P)}} \leq t \right)}{\partial t} = \frac{\partial \mathbb{P}\left(V_{T_L} \geq \frac{\theta}{kte^{-\rho(L-C_P)}} \right)}{\partial t}
\end{aligned}
$$

For $K(t) = \frac{\theta}{kte^{-\rho(L-C_P)}}$, we have

$$
\begin{aligned}
\mathbb{P}\left(\Delta T(k,L) \in dt\right) &= \frac{\partial \mathbb{P}\left(V_0 e^{\left(\mu-\frac{\sigma^2}{2}\right)T_L+\sigma B_{T_L}} \geq K(t) \right)}{\partial t} \\
&= \frac{\partial \mathbb{P}\left(-\frac{B_{T_L}}{\sqrt{T_L}} \leq d_2(t) \right)}{\partial t} = \frac{\partial N(d_2(t))}{\partial t} \\
&= \frac{1}{\sqrt{2\pi}} e^{\frac{-d_2(t)^2}{2}} \cdot \frac{1}{K(t)\sigma\sqrt{T_L}} \cdot \frac{\theta}{kt^2 e^{-\rho(L-C_P)}} \qquad (43)
\end{aligned}
$$

and where

$$d_2(t) = \frac{ln\left(\frac{V_0}{K(t)}\right) + \left(\mu - \frac{\sigma^2}{2}\right)T_L}{\sigma\sqrt{T_L}} \tag{44}$$

- Integral I_1c

By proceeding along the same lines as Integral I_1b, we have

$$\begin{aligned}
I_1c &= E_{\mathbb{P}} \int_{T_L+\Delta T(k,L)\wedge H^+_{L_1,D}\wedge T}^{H^+_{L_1,D}\wedge T} \widehat{DGDP}_u\,(1-k)\,e^{-ru}du \\
&= E_{\mathbb{P}} \int_0^{\frac{L_1-L}{a}+D} \int_{T_L+t}^{H^+_{L_1,D}\wedge T} \widehat{DGDP}_u\,(1-k)\,e^{-ru}du \cdot \mathbb{P}\left(\Delta T(k,L) \in dt\right)
\end{aligned}$$

Indeed, if $T_L + \Delta T(k, L)$ is higher than $\frac{L_1-L}{a} + D$, the integral is equal to zero.

$$\begin{aligned}
I_1c &= E_{\mathbb{P}} \int_0^{\frac{L_1-L}{a}+D} \int_{T_L+t}^{H^+_{L_1,D}\wedge T} V_0 e^{\left(\mu-\frac{\sigma^2}{2}\right)u+\sigma B_u} e^{-\rho(C_0+a(k)u-C_P)}\,(1-k)\,e^{-ru}du\,\cdot \\
&\quad \mathbb{P}\left(\Delta T(k,L) \in dt\right) \\
&= V_0\,(1-k)\,e^{-\rho(C_0-C_P)} E_{\mathbb{P}} \int_0^{\frac{L_1-L}{a}+D} \int_{T_L+t}^{H^+_{L_1,D}\wedge T} e^{(\mu-\rho a(k)-r)u} e^{-\frac{\sigma^2}{2}u+\sigma B_u}du\,\cdot \\
&\quad \mathbb{P}\left(\Delta T(k,L) \in dt\right) \\
&= V_0\,(1-k)\,e^{-\rho(C_0-C_P)} \Bigg\{ E_{\mathbb{P}} \int_0^{\frac{L_1-L}{a}} \int_{T_L+t}^{H^+_{L_1,D}\wedge T} e^{(\mu-\rho a(k)-r)u} e^{-\frac{\sigma^2}{2}u+\sigma B_u}du\,\cdot \\
&\quad \mathbb{P}\left(\Delta T(k,L) \in dt\right) \\
&\quad + E_{\mathbb{P}} \int_{\frac{L_1-L}{a}}^{\frac{L_1-L}{a}+D} \int_{T_L+t}^{H^+_{L_1,D}\wedge T} e^{(\mu-\rho a(k)-r)u} e^{-\frac{\sigma^2}{2}u+\sigma B_u}du \cdot \mathbb{P}\left(\Delta T(k,L) \in dt\right) \Bigg\}
\end{aligned} \tag{45}$$

Indeed, when $\Delta T(k, L)$ is smaller than $\frac{L_1-L}{a}$, the temperature process will never reach the tipping point L_1. In this case, the catastrophe is avoided and the upper bound of the second integral is T.

Applying Fubini's theorem and since $\left\{e^{-\frac{\sigma^2}{2}t+\sigma B_t}, t \geq 0\right\}$ is a *martingale*, we have

$$\begin{aligned}
I_1c &= V_0\,(1-k)\,e^{-\rho(C_0-C_P)} \\
&\quad \cdot \Bigg\{ \Bigg[\int_0^{\frac{L_1-L}{a}} \int_{T_L+t}^{T_L+t+\frac{L_1-(L+at)}{a(k)}+D} e^{(\mu-\rho a(k)-r)u}du \cdot \mathbb{P}\left(\Delta T(k,L) \in dt\right) \\
&\quad + \int_{\frac{L_1-L}{a}}^{\frac{L_1-L}{a}+D} \int_{T_L+t}^{T_L+\frac{L_1-L}{a}+D} e^{(\mu-\rho a(k)-r)u}du \cdot \mathbb{P}\left(\Delta T(k,L) \in dt\right) \Bigg] \cdot \mathbf{1}_{a(k)>0} \\
&\quad + \Bigg[\int_0^{\frac{L_1-L}{a}} \int_{T_L+t}^{T} e^{(\mu-\rho a(k)-r)u}du \cdot \mathbb{P}\left(\Delta T(k,L) \in dt\right) \Bigg] \cdot \mathbf{1}_{a(k)\leq 0}
\end{aligned}$$

$$+\left[\int_{\frac{L_1-L}{a}}^{\frac{L_1-L}{a}+D}\int_{T_L+t}^{T_L+\frac{L_1-L}{a}+D} e^{(\mu-\rho a(k)-r)u}du\cdot\mathbb{P}\left(\Delta T(k,L)\in dt\right)\right]\cdot\mathbf{1}_{a(k)\in[X,0]}$$
$$+\left[\int_{\frac{L_1-L}{a}}^{\frac{L_1-L}{a}+D}\int_{T_L+t}^{T} e^{(\mu-\rho a(k)-r)u}du\cdot\mathbb{P}\left(\Delta T(k,L)\in dt\right)\right]\cdot\mathbf{1}_{a(k)\in[-\infty,X]}\Bigg\} \tag{46}$$

Indeed, when $\Delta T(k, L)$ is higher than $\frac{L_1-L}{a}$, the catastrophe can still be avoided, if the time spent by the temperature process above the tipping point L_1 is smaller than D units of time, i.e. if $\Delta T(k, L) - \frac{L_1-L}{a} + \frac{L_1-(L-at)}{a(k)} \leq D$, i.e. if the new drift $a(k)$ is negative and small enough, that is:

$$a(k) \leq X \tag{47}$$

where:

$$X = \frac{L_1 - L - a\Delta T(k, L)}{D - \Delta T(k, L) + \frac{L_1-L}{a}} \leq 0 \quad \text{for} \quad X \in \left[\frac{L_1 - L}{a}, \frac{L_1 - L}{a} + D\right] \tag{48}$$

When $a(k)$ belongs to $[X, 0]$, the temperature drift is not small enough to avoid a catastrophe. The latter will occur at date $T_L + \frac{L_1-L}{a} + D$. If $a(k)$ is smaller than X, then the catastrophe will never occur. In this case, the GDP is maximized until time horizon T. Then

$$\begin{aligned}
I_1c = {} & \frac{V_0\,(1-k)\,e^{-\rho(C_0-C_P)}}{\mu-\rho a(k)-r} \\
& \cdot\Bigg\{\Bigg[\Bigg[\int_0^{\frac{L_1-L}{a}} e^{(\mu-\rho a(k)-r)\left(T_L+t+\frac{L_1-(L+at)}{a(k)}+D\right)}\mathbb{P}\left(\Delta T(k,L)\in dt\right) \\
& -\int_0^{\frac{L_1-L}{a}} e^{(\mu-\rho a(k)-r)(T_L+t)}\mathbb{P}\left(\Delta T(k,L)\in dt\right) \\
& +e^{(\mu-\rho a(k)-r)\left(T_L+\frac{L_1-L}{a}+D\right)}\mathbb{P}\left(\frac{L_1-L}{a}\leq\Delta T(k,L)\leq\frac{L_1-L}{a}+D\right) \\
& -\int_{\frac{L_1-L}{a}}^{\frac{L_1-L}{a}+D} e^{(\mu-\rho a(k)-r)(T_L+t)}\mathbb{P}\left(\Delta T(k,L)\in dt\right)\Bigg]\cdot\mathbf{1}_{a(k)>0} \\
& +\Bigg[e^{(\mu-\rho a(k)-r)T}\mathbb{P}\left(\Delta T(k,L)\leq\frac{L_1-L}{a}\right) \\
& -\int_0^{\frac{L_1-L}{a}} e^{(\mu-\rho a(k)-r)(T_L+t)}\mathbb{P}\left(\Delta T(k,L)\in dt\right)\Bigg]\cdot\mathbf{1}_{a(k)\leq 0} \\
& +\Bigg[e^{(\mu-\rho a(k)-r)\left(T_L+\frac{L_1-L}{a}+D\right)}\mathbb{P}\left(\frac{L_1-L}{a}\leq\Delta T(k,L)\leq\frac{L_1-L}{a}+D\right) \\
& -\int_{\frac{L_1-L}{a}}^{\frac{L_1-L}{a}+D} e^{(\mu-\rho a(k)-r)(T_L+t)}\mathbb{P}\left(\Delta T(k,L)\in dt\right)\Bigg]\cdot\mathbf{1}_{a(k)\in[X,0]}
\end{aligned}$$

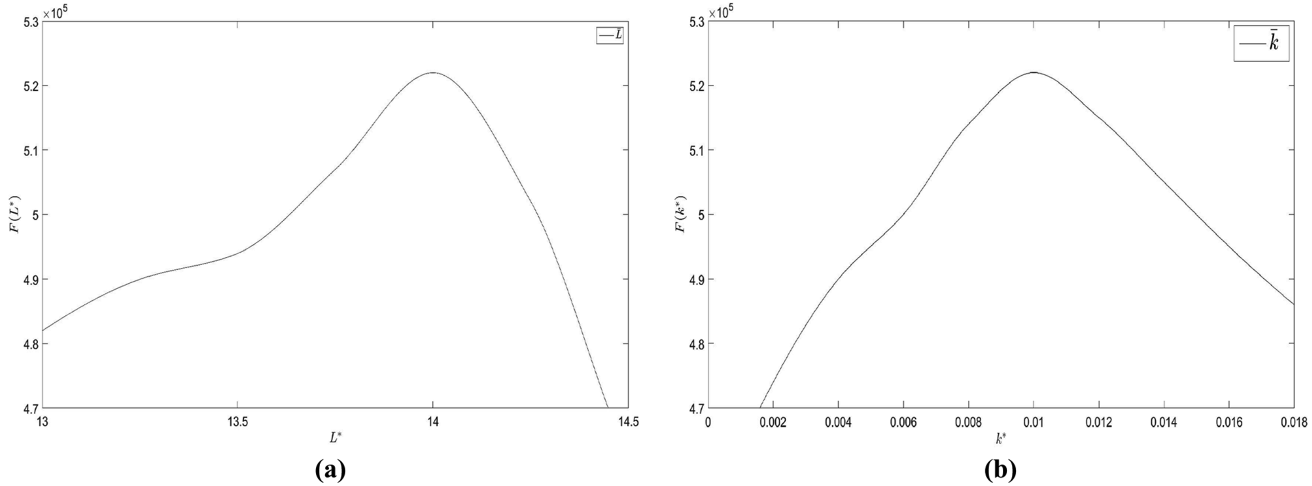


Fig. 9 Maximum of the function f, for $\beta = 0\,°\mathrm{C}$, plotted against the optimal investment threshold L^*, for $k^* = 1\,\%$, and against the optimal mitigation investment fraction k^*, for $L^* = 14\,°\mathrm{C}$

$$+\left[e^{(\mu-\rho a(k)-r)T}\mathbb{P}\left(\frac{L_1-L}{a}\leq\Delta T(k,L)\leq\frac{L_1-L}{a}+D\right)\right.$$
$$\left.-\int_{\frac{L_1-L}{a}}^{\frac{L_1-L}{a}+D}e^{(\mu-\rho a(k)-r)(T_L+t)}\mathbb{P}\left(\Delta T(k,L)\in dt\right)\right]\cdot\mathbf{1}_{a(k)\in[-\infty,X]}\Bigg\} \quad (49)$$

5.2 $L \geq L_1$

In order to solve Integrals I_2a, I_2b and I_2c, all the tools used to solve analytically the previous integrals have been applied.

- Integral I_2a

$$\begin{aligned}I_2a &= E_{\mathbb{P}}\int_0^{T_L\wedge H^+_{L_1,D}\wedge T} DGDP_u e^{-ru}du\\ &= V_0e^{-\rho(C_0-C_P)}\left\{E_{\mathbb{P}}\int_0^{T_L}e^{(\mu-\rho a-r)u}e^{-\frac{\sigma^2}{2}u+\sigma B_u}du\cdot\mathbf{1}_{T_L<H^+_{L_1,D}}\right.\\ &\quad\left.+E_{\mathbb{P}}\int_0^{H^+_{L_1,D}}e^{(\mu-\rho a-r)u}e^{-\frac{\sigma^2}{2}u+\sigma B_u}du\cdot\mathbf{1}_{T_L\geq H^+_{L_1,D}}\right\}\end{aligned}$$

As opposed to before, here when $T_L \geq H^+_{L_1,D}$, i.e., when $L \geq L_1 + aD$, then $H^+_{L_1,D} = \frac{L_1-C_0}{a} + D$, which is deterministic.

$$\begin{aligned}I_2a &= V_0e^{-\rho(C_0-C_P)}\left\{\int_0^{T_L}e^{(\mu-\rho a-r)u}du\cdot\mathbf{1}_{L<L_1+aD}\right.\\ &\quad\left.+\int_0^{H^+_{L_1,D}}e^{(\mu-\rho a-r)u}du\cdot\mathbf{1}_{L\geq L_1+aD}\right\}\\ &= \frac{V_0e^{-\rho(C_0-C_P)}}{\mu-\rho a-r}\\ &\quad\cdot\left[\left(e^{(\mu-\rho a-r)T_L}-1\right)\cdot\mathbf{1}_{L<L_1+aD}+\left(e^{(\mu-\rho a-r)(\frac{L_1-C_0}{a}+D)}-1\right)\cdot\mathbf{1}_{L\geq L_1+aD}\right]\end{aligned}$$

- Integral I_2b

  If L is high enough, the catastrophe occurs before the temperature level L is reached. In this case, the integral I_2b is equal to zero, then

$$I_2b = I_1b\cdot\mathbf{1}_{L<L_1+aD} \quad (50)$$

- Integral I_2c

  As in integral I_2b, if L is high enough then the catastrophe occurs before the temperature level L is reached. In this case, the integral I_2c is equal to zero, so

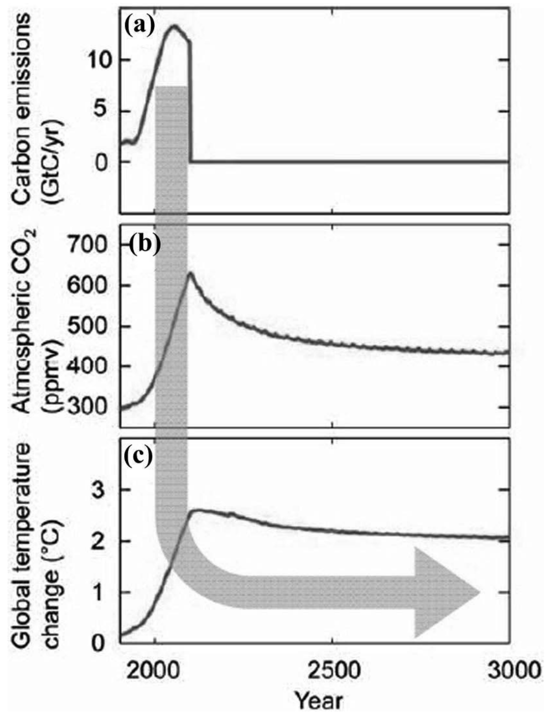
$$I_2c = I_1c\cdot\mathbf{1}_{L<L_1+aD} \quad (51)$$

In Figure 9, we shed light on the unicity of the solution when the temperature process is deterministic:

Appendix 4: Curbing emissions and its effects on future temperatures

As can be inferred from Fig. 10, it would take a long time after emissions are reduced to acceptable levels before the global mean temperature would reach its pre-industrialization level (Friedlingstein et al. 2011; IPCC 2013).

Fig. 10 Decay rate of temperature as a function of emission reductions



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Appendix D

Published Manuscript: *A Quantitative and Qualitative Analysis of the Super-efficient Equipment Program Subsidy in India*

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ORIGINAL ARTICLE

A quantitative and qualitative analysis of the super-efficient equipment program subsidy in India

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Abstract India is a fast growing economy with a fast growing population. Estimations indicate that in the next few years, it will be the world's most populous country, with China only second. These developments, together with high urbanization rates, are putting increasing pressure on the energy sector and driving the attention to energy efficiency. The issue is not only financial but also social and environmental. One of the schemes developed to promote energy efficiency in the residential housing sector is the Super Efficient Equipment Program (SEEP), which aims at reducing energy consumption in Indian households, by subsidizing the production of super efficient fans. Emphasis is put into increasing economies of scale and balancing possible losses in market power for producers. However, the scheme does not take into consideration consumers' behavior in the market and their purchasing propensity toward energy efficient fans. In this article, we develop an econometric model that takes consumers' preferences and behavior into consideration, by analyzing how these influence the success of the SEEP scheme. To do so, the price elasticity of energy efficient fans is calculated. We will then apply it to the SEEP subsidy scheme in order to assess how the quantity of super efficient fans sold varies with changes in the price. Using projected data on residential housing floor space in India, we will estimate the percentage of super efficient fans sold and calculate future energy savings. We are finally able to infer whether the SEEP scheme is capable of meeting its goals.

Keywords Energy efficiency · Energy efficient buildings · Green housing · Subsidy · Climate change · CO_2

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Introduction

Energy efficiency has drawn a lot of attention worldwide, being considered as one among the best and most effective mitigation solution to climate change. The spectrum of possibilities for this scope is wide and heterogeneous. Nevertheless, attention has usually been focused on the transportation and on the building sector. As both account for more than 20 % of global greenhouse gas emissions yearly (about 16 % the first, about 8 % the latter, excluding emissions from energy supply, which alone account for about

41 %), it is of particular importance to find adequate ways to direct mitigation (and later adaptation) efforts to these two sectors by the means of energy efficiency (IPCC 2014).

The building sector has, compared to others, high mitigation potential, while, at the same time, benefiting from a certain degree of cost-effectiveness. However, improving energy use in the building sector is not without difficulties. The problem is particularly evident in developing countries and specifically in Asia, where the urban population is growing at rates not experienced in the past (World Health Organization 2012). According to the Asian Development Bank (2012), the number of megacities (cities with population of more than 10 million individuals) will keep growing, the larger share to be found in the two most populous countries in the world, China and India. In light of this, the problem of sustainable buildings becomes of urgent solution.

Current GDP growth and increasing number of people moving from rural areas toward cities located in the most developed regions, imply a higher future demand for electricity. As an example, as income increases, the demand for air coolers increases (Phadke et al. 2014). In addition, global warming will obviously be itself an additional driver for higher electricity demand. The issue of energy efficiency for residential housing is particularly serious in India. According to UN projections, India will be the most populous country by 2060, hosting approximately 1.7 billion people (James 2011). For what concerns its impact on the environment, India, in 2014, placed itself third in the ranking of the biggest emitters, although per capita greenhouse gas emissions are among the lowest. Almost one third of the energy available in India is consumed by the residential and commercial building sectors. Kumar (2010) estimates that by 2030, the Indian commercial building stock, in square meters, will increase by 70 %. However, it is expected that residential buildings will grow at high rates as well. Bhattacharyya (2015) estimates that by 2030 the energy demand in the Indian residential sector will be four times higher than what it was in 2010. In light of these results, it is clear the extent of the energy burden India will have to bear, a big share of which originating from the residential building sector. Moreover, the differences between consumption in rural and urban India are still strong. Monthly per capita consumption of electricity in rural India is about 8.9 kWh per month, at an average cost of 25 Rupees ($0.38) per Kilowatt. Conversely, in urban India, an individual consumes on average 25.8 kWh of electricity per month, at a cost of around 87 Rupees ($1.33) per Kilowatt.[1]

In order to promote the diffusion of energy efficiency in construction methods and in the type of appliances produced and sold, the Government of India has developed some standards and some financing schemes. Among the first, we find the Standard and Labelling (S&L) Program and the Energy Conservation Building Code. The Standard and Labelling Program, launched in the Spring of 2005, applies energy scoring labels to many electrical appliances, such as refrigerators, air conditioners, LPG stoves, washing machines, color TVs, and ceiling fans. It is meant to provide costumers with an easy-to-read energy score in order to help them make an informed choice about the energy consumption of their desired appliance. It does not set any minimum requirement to the producers of such appliances, as the label is provided ex-post. Established in 2007, the Energy Conservation Building Code aims at revolutionizing the way buildings are constructed. It regulates lighting, cooling, the electrical systems, and also the way the building envelope has to be designed (Liu et al. 2010). For what concerns financing mechanisms, one of the most important ones, in terms of investments and goals, is the Super Efficient Equipment Program (SEEP). The SEEP scheme consists in a subsidy paid out to producers of electrical appliances, so to make the price of an energy efficient item similar to the price of the non-energy efficient ones. The idea is to promote the purchase and diffusion among households of energy-saving electrical appliances, so to obtain an avoided electricity capacity of 19,000 MW in the pilot phase of the program, i.e., the first 3 years (Climate Investment Funds 2013). During this phase, the program is oriented at subsidizing maximum Rp. 250, or $3.81, to the producers of the so called super efficient cooling fans, which have an energy consumption of 35 W, much lower than the most efficient fan currently sold in India, which on average consumes around 65 Watts. In other words, and thanks to this scheme, manufacturers can now have the opportunity

[1]Indian Ministry of Statistics and Programme Implementation, http://mospi.nic.in/Mospi_New/site/home.aspx

to "develop, produce, and sell super efficient equipment and appliances (SEE) at prices comparable to an average appliance" (Chunekar and Singh 2013). The total sum financed amounts to \$50,000,000 (Rp. 761,500) in the pilot phase of the program, and the main sponsors of the program is the Clean Technology Fund (Climate Investment Funds 2013).

As the Super Efficient Equipment Program looks very promising, the goal of our research will be to assess whether such subsidy scheme for energy efficiency is tackling the right entities, for the correct amount, and it is capable of obtaining the results sought after by the Government of India. Using data on household consumption, on demand of building and appliances, population projections and income, we will first find consumer preferences for energy efficient fans and then, using future electricity-use projections, we will determine whether this financing mechanism is capable of making a difference and how policies can drive households in knowing, choosing and taking advantage of the right incentives.

This article is structured in the following way. In the "Literature review" section, the current state of the literature on energy efficiency and energy demand in India is discussed. In the "Econometric assessment of the super efficient equipment program in India" section, the model is be presented and explained in details. In the "Future projections and future energy savings" section, we estimate the future demand for super efficient appliances. The "Results and discussion" section presents and discusses the results of the analysis. Finally, in the "Conclusion" section, some conclusions are drawn.

Literature review

Energy efficiency has always been considered as an effective cost saving solution to climate change (IPCC 2014). Since more than a decade, the scientific community and academia have started to study its development and applications. Sinha (2015) shows that energy efficiency can act as a driver of economic growth in India. Despite this, the diffusion of efficient appliances has been facing difficulties. Some structural barriers, common to many developing countries, have been identified in the literature, since their comprehension and analysis are important for a better design and management of policies and business practices. Reddy and Shrestha (1998) identify several barriers, such as lack of awareness, high costs, non-availability in the market of the desired efficient goods, uncertain savings over the long run and uninterested consumers. Hirst and Brown (1990) consider two main categories of barriers to the development of energy efficiency. The authors distinguish between structural and behavioral barriers. Among the first, they identify those elements out of the control of end-users, such as fuel prices, supply characteristics, difficult access to capital, and suboptimal regulatory policies. Among the behavioral characteristics, the authors identify misplaced incentives, individual attitude toward risk and also personal beliefs about energy efficiency. What makes the issue even more problematic are the interdependencies, at time obvious at times subtle, between the two groups and between the elements of the same group. In the category of non-behavioral barriers, we find an element common to many developing countries and to India particularly, that is *price distortion* (Gunatilake et al. 2014). Due to the fact that electricity is heavily subsidized, trades at very low cost and it is anyway too differentiated across regions, the signal that arrives to customers and to end users in general is not reflecting the correct price. In this setting, consumers cannot value correctly the trade off between electricity savings and the cost of implementing energy efficient measures. Other two important barriers that hinder the success of energy efficiency projects are *transaction costs* and *contract enforcement*. As shown by Shukla and Chaturvedi (2013), the first is a very problematic issue, and greatly influences the financial perspective and the diffusion of energy efficiency. Transaction costs are a problem in this setting not only for India but also for any other country and regard in the specific the residential building sector, where projects are developed on a (fairly) small scale.

Issues with contract enforcement are a fundamental element for any business environment. Among other consequences (lack of property rights, difficulty in forcing the contract to be executed, certainty of legal sentence in court), what this implies for corporate finance is a tendency, from companies, to vertically integrate. This mirrors their preference toward the internal development of projects, channeling resources internally rather than externally (Taylor et al. 2008). This could somehow lead to suboptimal business strategies for the firm. What could be done by external consultant or project developers with expertise in the field and (better) access to financing, will probably be done internally where specific knowledge is lacking, as it is often the cases in developing countries. Doing so, the probability of wasting financial resources for due diligence and for undertaking wrong investments is higher. The issue of weak contract enforcement is particularly true, as already mentioned, in developing countries and specifically in India, which, in the list of countries ranked by the rate of contract enforcing, positioned itself 184th over 185 countries. This weakness is also reflected by the *Ease to do Business* ranking, where India positioned 132nd over the same 185 Countries. Obviously, for not so widespread and not so mature business practices, as investments in residential energy efficiency could be, it is very hard to obtain financing and lead projects to completion. This issue is particularly severe as it influences the success of minimum standards and of financing schemes. In fact, as Phadke et al. (2014) have noted, the diffusion of energy efficiency in residential housing has yet to pick up momentum and these mechanisms have to be revised and improved significantly.

The presence of these barriers is one of the reasons why financing schemes such as subsidies are preferred. However, even without barriers, this kind of market interventions require a good understanding and a good estimation of business practices and of the purchasing attitude of the consumers (Liu et al. 2015). Some important studies have identified consumer behavior and purchasing propensity as important variables in the estimation of energy demand and its diffusion. For example, analyzing urban household survey data in India, Filippini and Pachauri (2004) find that electricity is a necessary good with a fairly inelastic demand, stable over three different climatic seasons, i.e., summer, winter, and monsoon season. There is no doubt that this has strong implications for policies aimed at reducing energy poverty and/or at increasing energy efficiency in India. Micro-level household survey data is used by Pachauri (2004) as well, who finds that energy demand in India is extremely dependent on the income level of the household. Moreover, the author finds that the age of the head of the household is a driver for energy consumption, while an higher literacy level of the household contributes to a more efficient energy usage. Both Filippini and Pachauri (2004) and Pachauri (2004) prove that understanding consumers' characteristics and behavior in estimating the demand for energy is a key aspect to keep in mind. Our analysis draws from these two articles and uses consumer preferences, in the form of price elasticities, in order to understand whether the Super Efficient Equipment Program (SEEP) is capable of transforming in the short term the demand for energy efficient appliances and consequently their market.

Econometric assessment of the super efficient equipment program in India

In India, households in urban and rural areas are growing in absolute numbers, as shown in Table 1. Changes in the average numbers of occupants per household are an important indicator of future consumption. If in the past only one item was needed for a certain number of people, now this number is decreasing, so that more items will be bought. As an example, if in urban India, in 2001, a refrigerator or a fan could be enough for slightly more than five people, now one refrigerator, on average, will satisfy the needs of less than four. Details can be seen in Table 2. At this stage, the question becomes: are existing financing schemes enough

Table 1 Indian population and households

Year	Population			Occupied residential buildings		
	Urban	Rural	Total	Urban	Rural	Total
2001	286,119,689 (27.82 %)	742,490,639 (72.18 %)	1,028,610,328 (100 %)	55,436,290 (28.77 %)	137,235,518 (71.23 %)	192,671,808 (100.00 %)
2011	377,105,760 (31.16 %)	833,087,662 (68.84 %)	1,210,193,422 (100 %)	99,046,223 (32.35 %)	207,116,576 (67.65 %)	306,162,799 (100.00 %)

Source: India Census Office

Table 2 Average occupants per household

Average number of occupants per household			
Year	Urban	Rural	Urban + Rural
2001	5.10	5.40	5.33
2011	3.80	4.02	3.95

to guide the Indian population toward a more sustainable consumption, mainly in terms of energy efficiency? In this section, the article provides empirical estimation of the impacts of introducing a price subsidy for the production of super efficient fans under the National Mission for Enhanced Energy Efficiency - Super Efficient Equipment Program (SEEP).

The analysis is conducted via an econometric approach. The price elasticity of energy efficient fans will be calculated and compared to the one of non-energy efficient ones. Additionally, the price elasticity of demand for urban and rural households will be calculated as well. This last estimation will make the projection of future energy consumption and savings possible. Data on prices, quantities, and other macroeconomic variables needed for the analysis have been obtained by Thomson Datastream for what concerns the electric fans' quantity and price data, the National Sample Survey Organization (NSSO) Rounds for what concerns disposable income and mean per capita expenditure data, and the Government of India for what concerns households' statistics.

One of the main issues when analyzing time series of cooling and/or heating appliances is the seasonality factor. A Centred Moving Average (CMA_t) method[2] has been used to de-seasonalize the amount of electric fans sold and the time series of prices. Having obtained a de-seasonalized time series of quantities and prices, it is now possible to specify the regression model, which is of the form:

$$\begin{aligned} lnQ_{i,t} &= \alpha + \gamma_1\, lnP_{i,t} + \gamma_2\, lnD_{HH,t} \\ &\quad + \gamma_3\, lnPOP_{HH,t} + \epsilon_t \end{aligned} \tag{1}$$

where:

- $i = NE$ or $i = EE$ if the electric fan is, respectively, a non-energy efficient or an energy efficient one.
- $Q_{i,t}$ is the quantity of electric fans of category i sold.
- $P_{i,t}$ is the price at time t of one electric fan of category i.
- $D_{HH,t}$ is the disposable income, per household, at time t.
- $POP_{HH,t}$ is the population per household.

A few words have to be spent on the variables chosen. $Q_{i,t}$ represents the amount of electric fans sold and de-seasonalized, as already mentioned, using the Centred Moving Average Method. Imports and Exports have been neglected since their balance in the last years has been almost zero (±0.1 %). $DI_{HH,t}$ represents the Disposable Income per Household. For the first analyses, it has been preferred to the Mean Capital Expenditure, which is usually the first choice in most of the literature. Different income classes have different expenditures and interpret goods differently. Moreover, the Disposable Income seems to be a more suitable macroeconomic variable, that tends to capture the whole household budget, before consumption decisions are taken. Last but not least, Mean Per Capita Expenditure is often obtained from surveys and estimations, which make this variable more subject to biases. All monetary quantities have been taken at constant prices, so to better capture supply/demand dynamics rather than inflationary ones.[3] This is particularly useful in a setting such as the Indian one, where inflation in the past years has been showing very high levels. Finally, Population per Household has been chosen among the independent variables, instead of floor space, as it seems more descriptive of the consumption behavior of the different households, in particular when floor space data is scarcely available.

Here, an arbitrary distinction between government/commercial and private households has been made. In order to obtain a better estimation of private demand for energy efficient fans, the amount, in square meters, of floor space belonging to government

[2]The Centred Moving Average is a method used to de-seasonalize time series. Analytically, it is of the form:

$$CMA_t = \frac{y_{t-\frac{m}{2}} + 2\left(y_{t-\frac{m}{2}+1} + \cdots + y_t + \cdots + y_{t-\frac{m}{2}-1}\right) + y_{t-\frac{m}{2}}}{2m} \tag{2}$$

where $m = 10$ years represents the length of the moving average and y_t the observation at time t.

[3]To be precise, in order to take inflation away from the time series, the minimum between the inflation rate and the Consumer Price Index for electric fans has been removed from the price.

Table 3 Non-energy efficient fans, summary of statistics

Variable	Coefficient	(Std. Err.)	t-stat	(p value)
α	9.7228**	(0.7188)	13.53	(0.000)
$lnP_{NE,t}$	$\gamma_1 = -0.6798$**	(0.0941)	−7.22	(0.000)
$lnDI_{HH,t}$	$\gamma_2 = -0.0649$	(0.0648)	−1.00	(0.318)
$lnPop_{HH,t}$	$\gamma_3 = -1.1281$**	(0.1321)	−8.54	(0.000)
$N = 142$				
$F_{(3,138)} = 574.4$				

Significance levels: † 10 %, * 5 %, ** 1 %

and commercial buildings has been taken into consideration[4] and subtracted from the total. Afterwards, we will use the average cooling capacity in square meters of a ceiling fan and its ownership rates among households to determine how many fans are demanded in the residential sector.

Within the residential sector, some care has to be taken when considering the distribution of items across different households. For what concerns electric fans, one important aspect to be kept into consideration is the fact that different climatic regions show different supply and demand patterns. However, since urbanization is growing more in warmer regions, it is not a loss of generality to assume that the distribution of electric fans and other appliances will closely follow the growth in number of residential houses. Another issue to be considered is that rural and urban households have different behavior on the market. The diffusion of electrical appliances is still very biased toward urban areas, whereas rural populations rely on more traditional means.[5]

In addition, the *turnover* ratio of fans has to be kept into consideration, since it has strong impacts on the demand of electrical appliances.

The informal sector captures an important share of the market for electric appliances, traditional fans in particular. While we assume that occupants of new residential buildings avoid looking at the informal sector, we will release this assumption when analyzing the market for already existing, non-renovated buildings. The methodology used in this case will be presented in the "Estimation of the market for existing buildings" section.

All variables have been tested for stationarity and modified accordingly, using the method of seasonal differences. Additional tests of normality and cointegration have been run as well. Results for such tests can be found in Appendix 2.

Table 3 summarizes the statistical results obtained for non-energy efficient fans.

As can be noticed by looking at Table 3, γ_1, the price elasticity of demand for non-energy efficient fans, is above −1, i.e., $\gamma_1 = -0.6798$. This implies that demand is fairly inelastic. In other words, a 1 % decrease in the price of this category of fans will cause an increase of 0.6798 % in the amount of items demanded. Unsurprisingly, the amount of people per household shows negative elasticity, meaning that the lower the number of people sharing the same floor space, the more appliances are needed. Conversely, the disposable income per household shows no statistical significance ($p = 0.318$). This is a surprising result. The interpretation is that regular, non efficient fans are seen as primary goods by many customers, in a warm and humid country such as India. Their demand

Table 4 Energy efficient fans, summary of statistics

Variable	Coefficient	(Std. Err.)	t-stat	(p value)
α	15.2663	**(1.0567)	14.45	(0.000)
$lnP_{E,t}$	$\beta_1 = -0.9580$	**(0.1421)	−6.74	(0.000)
$lnDI_{HH,t}$	$\beta_2 = 0.5087$	**(0.0698)	7.29	(0.000)
$lnPop_{HH,t}$	$\beta_3 = -1.0822$	**(0.1228)	−8.81	(0.000)
$N = 142$				
$F_{(3,138)} = 2554.9$				

Significance levels: † 10 %, * 5 %, ** 1 %

[4]As of end 2012, Government and Commercial Buildings accounted for about 800 million sq.m of floor space. Source: USAID ECO-III Project.

[5]As an example, 81.8 % of urban households and 34.8 % of rural households have an electric fan, as of 2012 (Chaturvedi et al. 2014b).

Table 5 Summary of elasticities for rural and urban households

Variable	Coefficient	Value	p value
Rural			
$lnP^R_{NE,t}$	β^R_1	−0.2514	(0.000)
$lnP^R_{E,t}$	β^R_1	−0.8075	(0.000)
Urban			
$lnP^U_{NE,t}$	β^U_1	−0.1869	(0.000)
$lnP^U_{E,t}$	β^U_1	−0.6268	(0.000)

is not determined by the disposable income, but by customers' needs.

In order to assess the price elasticity of energy efficient fans, their quantity has been regressed against their average price, the disposable income per households and the population per household, all taken in logarithmic form as before. Table 4 shows the results obtained by our regression analysis.

Here, β_1, the price elasticity of demand for energy efficient fans, is very close to -1. Price is now a determining factor for quantity demanded. A 1 % decrease in the price of an energy efficient fan causes an increase in the demand for the same good of almost equal amount, i.e., $\beta_1 = -0.9580$. Population per household shows negative elasticity as before, and the same conclusions can be drawn. In contrast, disposable income is now statistically significant and shows a coefficient $\beta_2 = 0.5087$. This independent variable is now much more important in explaining the purchase propensity of efficient fans. Our results suggest that energy efficient fans are not considered primary goods as regular fans, and their purchase depends primarily on income rather than on necessity. It also proves that energy efficient fans still fall in the category of niche, middle-high class products.

As mentioned, India presents strong differences in income, and also in appliance diffusion, between rural and urban areas. While the average disposable income in rural India was about Rs. 4645.19 per Month as of 2013, the urban population had an average monthly disposable income 53 % higher, about Rs. 7135.04. On the same level, Mean Per Capita Expenditure for durable goods in urban India, as of 2013, was 24.4 % higher than in rural India (Rs. 68.43 vs. Rs 55.01). As it is expected that more is spent in rural India for primary goods rather than for "leisure" goods, it can be inferred that urban India has more income available for non primary expenditures. As an example, the market penetration of ceiling fans in urban areas was 81.8 % as of 2013, while in rural India was 34.8 %. It is also true, however, that rural India holds the biggest share of total Indian disposable income, about 55 %, but it hosts about 70 % of the population.

In order to assess what is the impact of the Super Efficient Equipment Program, the above model has been applied, for the two categories of electric fans, to the two categories of households, rural and urban. Differently than before, as already mentioned, it is not possible to use the disposable income as an independent variable in this analysis. In fact, no separate disposable income time series data on rural and urban India is available. In order to overcome this issue, the Mean Per Capita Expenditure for durable goods, obtained by the NSSO Rounds, has been used. These data are available separately for urban and rural households. Equation 3 expresses the linear regression models used to estimate price elasticities for rural and urban households, and for non-energy efficient and energy efficient fans.

$$lnQ^j_{i,t} = \alpha + \eta^j_1 lnP^j_{i,t} + \eta^j_2 lnMPCE^j_t + \eta^j_3 lnPOP^j_t + \epsilon_t \qquad (3)$$

where $i = NE$ or $i = EE$ if the electric fan is, respectively, a non-energy efficient or an energy efficient one, and where $j = R$ or $j = U$ if the electric fan belongs, respectively, to a rural or to an urban household. The regression results are summarized in Table 5.

Table 6 Estimated existing fan stock

Household type	Ownership rate	Household with cooling fan	Total fans informal + formal	Total fans formal sector only
Urban	$O_U = 81.8$ %	52,994,112	158,982,338	39,745,584
Rural	$O_R = 34.8$ %	36,364,817	72,729,635	18,182,408

In general, these results confirm the ones obtained previously. However, having differentiated price elasticities between urban and rural households will allow us to predict future consumption of energy efficient fans among the two groups, as done in the "Results and discussion" section.

Future projections and future energy savings

Having found the price elasticities of demand, it is interesting to apply the results obtained to the future diffusion of energy efficient fans, making use of ownership rates and relying on the estimated growth rate of floor space in the next few years in India (Kumar 2010). Other than being able to assess how the market diffusion of these items changes when the price moves, the following analysis allows to estimate the future potential savings in terms of energy consumption. The forecast covers a time interval of 3 years, that is the pilot phase of the SEEP (Climate Investment Funds 2011). While the effective subsidy will be implemented using an auctioning mechanisms with multiple winners, according to the Super Efficient Equipment Program the maximum subsidy that producers can apply to the sale of energy efficient fans is Rs. 250, or \$3.81 (Climate Investment Funds 2013). The projected savings have to be compared to the baseline level ($S = 0$), so to see if the goal of an avoided electricity capacity of 1900 MW in 3 years, as foreseen by the Bureau of Energy Efficiency (Garnaik 2011) is reachable. The remaining assumptions are:

- Price elasticity of demand remains constant between year 1 (Y_1) and year 3 (Y_3).
- Producers produce super efficient fans always according to the minimum energy efficiency requirements of the SEEP Program, whose guidelines state that such fans should not use more than 35W of power.[6]
- Non-energy efficient fans use approximately 65 W of power.
- There are no intermediate agents in the market for electric fans.[7]
- Producers apply a price reduction of 100 % of the subsidy to the final good[8]

Table 7 Annual purchases per different household type and different fans

Urban households	Rural households
$MP_{U,RR} = 3,354,888$	$MP_{R,RR} = 1,534,896$
$MP_{U,NR} = 642,432$	$MP_{R,NR} = 293,892$

Estimation of the market for existing buildings

While it is known and clear that India's real estate market is booming now and will keep booming in the next few years, when discussing about the impacts of energy efficiency financing on energy consumption, existing households have to be considered in the calculations as well. The survival rate of electrical appliances, and of ceiling fans in our particular case, is very important in estimating how many of these appliances will be replaced on average, every year. This number has to be summed up to the new appliances needed for new constructions, so to give a complete estimation of how many appliances are sold and what their energy impact will be. The inclusion of existing appliances requires an estimation of the lifetime of this appliances. In order to do so, we make use of a Weibull distribution, which is very helpful for estimating how many appliances survive after years of use. The literature dealing with the estimation of cooling fans, both commercial and domestic is not extensive. However, important articles have shed light on the problem. Examples of the application of a Weibull distribution for the estimation of the average life of a fan can be found in Claassen et al. (1996) and in Mendenhall and Sincich (2007). For a more theoretical approach, please refer to Brown (1980). The Weibull distribution has a *probability density function* of the form $f(t) = \frac{\beta}{\alpha}\left(\frac{t}{\alpha}\right)^{\beta-1} e^{-\left(\frac{t}{\alpha}\right)^{\beta}}$, where, α is the scale parameter, while t is the so called age of the function, such that $f(t)dt$ is the probability that an appliance fails in the time interval dt. β is the so called shape parameter, which determines how failures are distributed. The shape parameter β can be smaller, equal, or grater than one, depending on whether the failure rate decreases,

[6]As there is a cap to the maximum subsidy obtainable by a producer, there will be no incentive to bear extra production costs.

[7]This assumption is also made by the SEEP Scheme.

[8]Meaning that no fraction of the subsidy is used to reduce costs that are not related to the production of energy efficient fans

Table 8 Summary of cumulative projected avoided electricity capacity (in MWh) for urban households (compared to baseline $S = 0$) and for existing fans only

Urban households Subsidy (Rs.)	Year	% EE fans (w.r.t BAU= 17.80 %)		% Avoided capacity in MWh	Avoided capacity in MWh
$S = 220$	1	18.09 %	(+1.63 %)	0.15 %	730.3
	3	19.99 %	(+12.30 %)	1.10 %	14,940.1
$S = 250$	1	18.38 %	(+3.25 %)	0.29 %	1460.6
	3	20.29 %	(+13.98 %)	1.25 %	16,945.7
$S = 300$	1	18.86 %	(+5.95 %)	0.53 %	2669.4
	3	20.79 %	(+16.79 %)	1.50 %	20,348.5
$Q_{TOT,Urb,Y_1} = 3,997,320$					$MWh_{Urb,BAU,Y_1} = 500,808.2$
$Q_{TOT,Urb,Y_3} = 10,802,412$					$MWh_{Urb,BAU,Y_3} = 1,353,391.0$

remains constant, or increases with time. While most of the literature uses $\alpha = 1$, here it has been chosen to be $\alpha = 0.874$ (Kim et al. 1996), so to take into account possible lower quality controls, humidity, faulty electric lines, and so on. In addition, a $\beta > 1$ is considered, i.e., $\beta = 1.5$ so that the probability of failures happening increases with the age of the appliance. The shape parameter also determines the *failure rate*, or *hazard function*, $h(t) = \frac{\beta}{\alpha}\left(\frac{t}{\alpha}\right)^{\beta-1}$.

Given these parameters, it is now possible to calculate the so called L_{10} value, which indicates the 10th percentile of the Weibull distribution. In other words, L_{10} represents the age at which 90 % of the fans is still in operation, and, among producers, it is a popular indicator of the life of an electrical appliance. Mathematically:

$$L_{Q=10} = \alpha \left[\int_0^t h(u)du\right]^{\frac{1}{\beta}} \approx 0.19497 \tag{4}$$

With $\alpha = 0.874$ and $\beta = 1.5$, the average hourly lifetime of a ceiling fan is approximately $L_{10} = 19,497$ h. Assuming that a ceiling fan is used on average for 7 h every day and for 300 days a year (Chaturvedi et al. 2014b), which is an assumption that takes into account that there could be variabilities in fan usage due to climatic variations, the average life of a ceiling fan in years has been estimated to be $L_{10}^{y} = 9.28$ years.

However, as said before, not all of the ceiling fans will break after L_{10} = 19,497 h of use, and not all of them which brake will be replaced by new ones. This last aspect represents one additional issue that has to be taken into account. In fact, in many developing Countries, as well as in many less developed regions of the OECD ones, the so called informal and secondary markets capture an important share of demand for electrical appliances, which are many times refurbished items. This makes it difficult to keep track of the number of sales. Such sectors cannot be analyzed in terms of the SEEP Program. In fact, it can be inferred that demand is satisfied by local shops whose sales of energy-efficient items are almost impossible to track. Consequently, the impact of a subsidy scheme in this "part" of the marked cannot be analyzed. Since it is estimated that 75 % of the Indian economy is captured by the informal and secondary sectors (Nataraj 2011; Chakrabarti 2013), the demand belonging to this percentage of households has to be subtracted from the total demand for ceiling fans. While excluding completely this share of demand for super efficient appliances may seem wrong, we have to think of the fact that secondary and informal markets need some time to develop. In our case, it is hard to imagine a significant presence of SEEP super efficient fans in secondary and informal markets already in the few years of the pilot phase of the program. Moreover, the informal and secondary markets are dominated by traders who deal with older, often re-assembled, appliances

Table 9 Summary of cumulative projected costs of the SEEP for urban households (compared to baseline $S = 0$) and for existing fans only

Urban households Subsidy (Rs.)	Year	% EE fans (w.r.t BAU= 17.80 %)	Absolute avoided capacity in MW	Cost of subsidy in Rp./USD
$S = 220$	1	18.09 % (+1.63 %)	0.35	159,085,341.36/2,422,822.02
	3	19.99 % (+12.30 %)	7.10	475,068,474.94/7,235,150.35
S = 250	1	18.38 % (+3.25 %)	0.70	183,676,854.00/2,797,343.38
	3	20.29 % (+13.98 %)	8.07	547,952,348.70/8,345,149.88
$S = 300$	1	18.86 % (+5.95 %)	1.27	226,168,365.60/3,444,476.36
	3	20.79 % (+16.79 %)	9.69	673,746,436.44/10,260,956.10
$Q_{TOT,Urb,Y_1} = 3,997,320$				$MWh_{Urb,BAU,Y_1} = 500,808.2$
$Q_{TOT,Urb,Y_3} = 10,802,412$				$MWh_{Urb,BAU,Y_3} = 1,353,391.0$
USD/INR=Rp. 65.6612				
INR/USD=\$0.01523				

(Fernando 2009) or who dismember the appliances themselves in order to sell the single components, as it happens with LEDs, light emitting diodes, or CFLs, compact fluorescent lamps (Chaturvedi et al. 2014a).

One other subtraction has to be made. In fact, of the total existing 244,641,582 households, around 75,360,000 have no electricity, so it is fair to say that members of these households will not enter the market for new fans. This number comprehends also those households belonging to the poorest urban areas, around 13,700,000.[9] But how many ceiling fans are needed, on average, to cool down residential buildings? In India, around 800 million m^2 of floor space belong to government and commercial buildings, while approximatively 10 billion m^2 of floor space belong to residential buildings (Manu et al. 2011). This implies that each household occupies, on average, 40.88 m^2 of space. Given that a ceiling fan with a 120-mm blade cools down around 13 m^2 of space, net of furniture, we can infer that each household has three fans, on average. While we will keep this estimation for the urban sector, being it in line with the recent literature (Chaturvedi et al. 2014b), for the rural sector we will again rely on Chaturvedi et al. (2014b), which consider the rural sector as possessing, on average, two fans per household.[10] While it may seem unrealistic that rural India possesses such high quantity of fans, it has to be understood that electric fans are very cheap and among the first appliances households buy, as intuitively described by van Ruijven et al. (2011).

Now, in order to be able to use the number of ceiling fans per household closest as possible to reality, we will use fan ownership rates as found in Chaturvedi et al. (2014b). The ownership rate for urban and rural India are respectively, $O_U = 81.8$ % and $O_R = 34.8$ %. Given all the available data on floor space, the number of households and the estimated amount of ceiling fans per household, we can finally find the stock of fans. The results are visible in Table 6.

In addition, rarely in these realities one electrical appliance is replaced by a new one once it stops functioning. To make our analysis more realistic, we assume that 85 % of the broken items can be repaired, which is a percentage in line with the average reparation rate of electrical appliances. In order to see how long a repaired electrical appliance lasts, the so called

[9]Ministry of Statistics of India, 2011

[10]While being in line with our findings for what concerns the urban households.

Table 10 Summary of cumulative projected avoided electricity capacity (in MWh) for rural households (compared to baseline $S = 0$) and for existing fans only

Rural households Subsidy (Rs.)	Year	% EE Fans (w.r.t BAU= 17.80%)		% Avoided capacity in MWh	Avoided capacity in MWh
$S = 220$	1	18.69 %	(+5.00 %)	0.45 %	1018.6
	3	20.62 %	(+15.84 %)	1.42 %	8721.9
$S = 250$	1	19.07 %	(+7.13 %)	0.64 %	1453.5
	3	21.01 %	(+18.03 %)	1.61 %	9928.1
$S = 300$	1	19.68 %	(+10.56 %)	0.96 %	2151.6
	3	21.65 %	(+21.63 %)	1.94 %	11,907.6
$Q_{TOT,Rur,Y_1} = 1,816,644$					$MWh_{Rur,BAU,Y_1} = 227,600.1$
$Q_{TOT,Rur,Y_3} = 4,909,320$					$MWh_{Rur,BAU,Y_3} = 615,069.1$

Mean Time Between Failures has to be calculated. The MTBF is (Zacks and Even 1966; Tavner et al. 2007):

$$\text{MTBF} = \alpha \cdot \Gamma(1 + \frac{1}{\beta}) \approx 0.874 \cdot \Gamma(1.667) = 0.7889 \tag{5}$$

where $\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}dt$ represents the Gamma Function. The Mean Time Between Failures (MTBF) is very useful in this context since a repaired or refurbished electrical appliance does not last as long as a new one, on average. The MTBF tells us that repairing/refurbishing an appliance increases its lifetime by 0.7889 years. In other words, maintenance extends lifetime by 10 %, on average. The remaining 15 % of fans that cannot be repaired, or whose reparation costs are too high, leave the owner with no other choice but replacing it with a new one. Having found what the

Table 11 Summary of cumulative projected costs of the SEEP for rural households (compared to Baseline $S = 0$) and existing fans only

Rural households Subsidy (Rs.)	Year	% EE Fans (w.r.t BAU= 17.80 %)		Avoided capacity in MW	Cost of subsidy in Rp./USD
$S = 220$	1	18.69 %	(+5.00 %)	0.48	74,696,767.99/1,137,609.37
	3	20.62 %	(+15.84 %)	4.15	201,861,419.76/3,074,288.86
S = 250	1	19.07 %	(+7.13 %)	0.69	86,608,502.70/1,319,021.51
	3	21.01 %	(+18.03 %)	4.72	234,051,831.00/3,564,539.17
$S = 300$	1	19.68 %	(+10.56 %)	1.02	107,254,661.76/1,633,456.32
	3	21.65 %	(+21.63 %)	5.67	289,846,252.80/4,414,271.48
$Q_{TOT,Rur,Y_1} = 1,816,644$					$MWh_{Rur,BAU,Y_1} = 227,600.1$
$Q_{TOT,Rur,Y_3} = 4,909,320$					$MWh_{Rur,BAU,Y_3} = 615,069.1$
USD/INR=Rp. 65.6612					
INR/USD=$0.01523					

Table 12 Summary of cumulative projected avoided electricity capacity (in MWh) for all households (compared to Baseline $S = 0$) and for existing fans only

All Households				
Subsidy (Rs.)	Year	% EE Fans	% Avoided Capacity	Avoided Capacity
		(w.r.t BAU= 17.80 %)	in MWh	in MWh
$S = 220$	1	18.28 % (+2.68 %)	0.24%	1748.9
	3	20.19 % (+13.43 %)	1.20 %	23,662.0
S = 250	1	18.60 % (+4.47 %)	0.40 %	2914.1
	3	20.52 % (+15.25 %)	1.37 %	26,873.6
$S = 300$	1	19.12 % (+7.39 %)	0.65 %	4821.0
	3	21.06 % (+18.31 %)	1.64 %	32,256.8
$Q_{TOT,AllHH,Y_1} = 5,813,964$				$MWh_{AllHH,BAU,Y_1} = 728,408.3$
$Q_{TOT,AllHH,Y_3} = 15,711,732$				$MWh_{AllHH,BAU,Y_3} = 1,968,460.1$

average age of a fan is in years, it is now possible to estimate which is the number of purchases/repairs needed. We also distinguish between Urban and Rural Households, and also between repairable (RR) and not repairable (NR) items. Following Taylor and Fujita (2012), the annual purchases of non repairable appliances are so calculated:

$$AP_{i,NR} = \frac{\text{average stock}}{\text{lifetime}} \tag{6}$$

while the annual purchases of the repairable appliances are so calculated:

$$AP_{i,RR} = \frac{\text{average stock}}{\text{lifetime + MTBF}} \tag{7}$$

where $i = U, R$, depending on whether the household under consideration belongs to urban or rural India. Results can be seen in Table 7.

Table 13 Summary of cumulative projected costs of the SEEP for all households (compared to baseline $S = 0$) and for existing fans only

All households				
Subsidy (Rs.)	Year	% EE Fans	Avoided capacity	Cost of subsidy
		(w.r.t BAU= 17.80 %)	in MW	in Rp./USD
$S = 220$	1	18.28 % (+2.68 %)	0.84	233,782,109.4/3,560,431.4
	3	20.19 % (+13.43 %)	11.27	676,929,894.7/10,309,439.2
$S = 250$	1	18.60 % (+4.47 %)	1.39	270,285,356.7/4,116,364.9
	3	20.52 % (+15.25 %)	12.79	782,004,179.7/11,909,689.1
$S = 300$	1	19.12 % (+7.39 %)	2.29	333,423,027.4/5,077,932.7
	3	21.06 % (+18.31 %)	15.37	963,592,689.2/14,675,227.6
$Q_{TOT,AllHH,Y_1} = 5,813,964$				$MWh_{AllHH,BAU,Y_1} = 728,408.3$
$Q_{TOT,AllHH,Y_3} = 15,711,732$				$MWh_{AllHH,BAU,Y_3} = 1,968,460.1$
USD/INR=Rp. 65.6612				
INR/USD=$0.01523				

Table 14 Summary of cumulative projected avoided electricity capacity (in MWh) for all households (compared to baseline $S = 0$), and for existing and new fans

All households Subsidy (Rs.)	Year	% EE Fans (w.r.t BAU= 17.80 %)	% Avoided capacity in MWh	Avoided capacity in MWh
$S = 220$	1	18.15 % (+1.97 %)	0.18 %	4689.3
	3	20.05% (+12.64 %)	1.13 %	81,465.8
$S = 250$	1	18.44 % (+3.59 %)	0.32 %	8574.8
	3	20.35 % (+14.33 %)	1.28 %	92,327.9
$S = 300$	1	18.93 % (+6.35 %)	0.57 %	15,139.8
	3	20.86 % (+17.19 %)	1.54 %	110,793.5
$Q_{TOT,AllHH,Y_1} = 21,266,796$				$MWh_{AllHH,BAU,Y_1} = 2,664,431.8$
$Q_{TOT,AllHH,Y_3} = 57,471,477$				$MWh_{AllHH,BAU,Y_3} = 7,200,371.5$

Results and discussion

In order to estimate future energy savings obtainable after the introduction of a subsidy scheme such as the Super Efficient Equipment Program (SEEP) in India, we take results from Table 5 and apply them to the future demand of energy efficient fans. We extend our analysis by adding those fans that are already existing and that might be replaced, once broken, by energy-efficient fans. To do so, we combine the results in Table 5 and with the results obtained in the "Results and discussion" section. We also introduce the "Cost of Subsidy," which represents the amount of financing effectively needed to replicate the results in the following tables. We start by analyzing, separately, the urban and the

Table 15 Summary of cumulative projected costs of the SEEP for All Households (compared to Baseline $S = 0$), and for Existing and New Fans

All households Subsidy (Rs.)	Year	% EE fans (w.r.t BAU= 17.80 %)	Avoided capacity in MW	Cost of subsidy in Rp./USD
$S = 220$	1	18.15 % (+1.97 %)	2.23	849,183,164.28/12,932,804.84
	3	20.05 % (+12.64 %)	38.79	2,294,836,076.61/34,949,665.00
$S = 250$	1	18.44 % (+3.59 %)	4.08	980,399,295.60/14,931,187.15
	3	20.35 % (+14.33 %)	43.97	2,649,435,089.70/40,350,101.59
$S = 300$	1	18.93 % (+6.35 %)	7.21	1,207,714,344.84/18,393,538.36
	3	20.86 % (+17.19 %)	52.76	3,596,656,030.66/54,774,606.45
$Q_{TOT,AllHH,Y_1} = 21,266,796$				$MWh_{AllHH,BAU,Y_1} = 2,664,431.8$
$Q_{TOT,AllHH,Y_3} = 57,471,477$				$MWh_{AllHH,BAU,Y_3} = 7,200,371.5$
USD/INR=Rp. 65.6612				
INR/USD=0.01523				

rural sectors. We will then combine them to be able to find the future energy savings for all households, first for the stock of existing fans only, and then including also our estimation on the "consumption" of new fans.

One additional remark has to be made. While the maximum obtainable discount, according to the scheme's guidelines (Climate Investment Funds 2013), is Rp. 250, or $3.81 (which corresponds to the maximum amount given as a subsidy to producers), we tested also what the results could be in case the discount would be lower, i.e., in case the producers decide to keep part of the subsidy for themselves, or in case the discount would be higher, i.e., in case some economies of scale or other factors start playing a role and the producers are able to apply a bigger discount to the price of energy efficient fans.

Urban households

Results regarding energy savings of existing fans in urban households are visible in Tables 8 and 9. The discount of Rp. 250 brings an avoided electricity capacity of 1.25 %, that translates into avoided 8.07 MW in year 3. When increasing the discount to Rp. 300, we obtain an avoided electricity capacity addition of 1.50 %, that translates into avoided 9.69 MW. Conversely, when reducing the discount to Rp. 220, we obtain an avoided electricity capacity of 1.10 %, or 7.10 MW. In year 3, the market share of energy efficient fans is, respectively, 19.99, 20.29, and 20.79 % for the Rp. 220, the Rp. 250 and for the Rp. 300 discounts. Compared to the Business-as-Usual case (market share of 17.80 %), it means an increase of 12.30 %, of 13.98 %, and of 16.79 %, respectively. Regarding the cost of the Rp. 250 subsidy, in order to obtain such results the amount to be financed would be of approximately Rp. 547,952,348.70, or $8,345,149.88. For the Rp. 220 and Rp. 300 subsidies, the costs would approximately amount to Rp. 475,068,474.94, or $7,235,150.35, and to Rp. 673,746,436.44, or $10,260,956.10.

Rural households

Despite giving home to the biggest share of Indian population, rural households account for approximately half the demand for fans. Lower income and a more difficult access to markets (Balachandra 2011) are among the key issues. However, our results are unexpected. In comparison to urban households, rural households show a higher propensity to the purchase of energy-efficient fans,

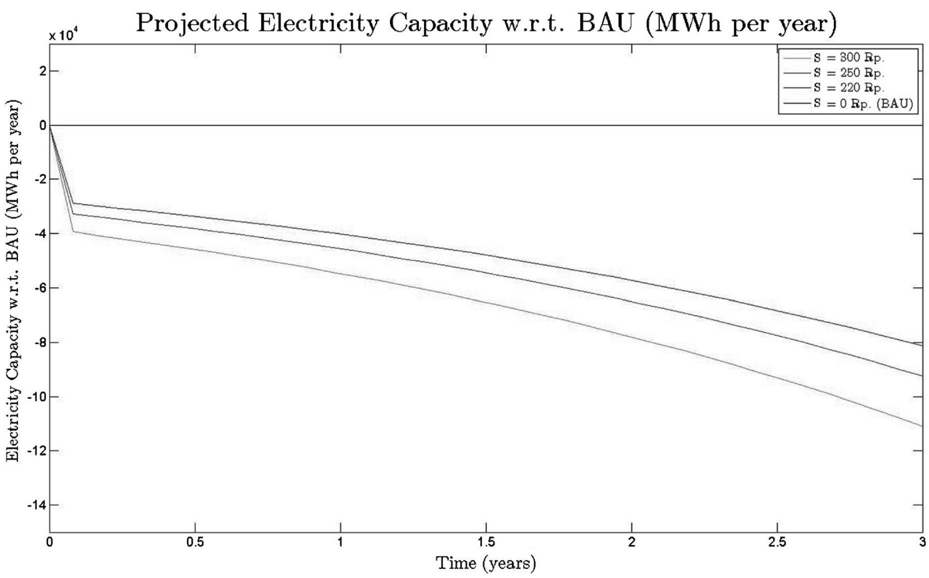

Fig. 1 Projected future electricity capacity from existing and new electric fans in all households

following the results in Table 5. The Rp. 220, Rp. 250, and Rp. 300 discounts achieve, respectively, 1.42, 1.61, and 1.94 % in avoided electricity capacity. The costs of the scheme sum up to, respectively, Rp. 201,861,419.76 ($3,074,288.86), Rp. 234,051,831.00 ($3,564,539.17), and Rp. 289,846,252.80 ($4,414,271.48). Market shares (and percentage increase w.r.t the Business-as-Usual case) become respectively 20.62, 21.01, and 21.65 %. What is interesting is that the scheme seems to be more successful, and economically efficient, in rural rather than in urban households. As an example, the cost of avoiding 1 W[11] of electricity capacity, for the Rp. 250 subsidy, is Rp. 50 (or $0.76) in the rural sector and Rp. 67.9 (or $1.03) in the urban sector. This means that avoiding electricity capacity in urban areas costs about 36 % more than what it costs in the rural ones (Tables 10 and 11) .

All households, existing fans

In Tables 12 and 13, the combined total avoided electricity capacity for urban and rural households, for existing fans only, are shown. Total avoided electricity capacities achieved are 1.20 % for the Rp. 220 subsidy, 1.36 % for the Rp. 250 subsidy, and 1.64 % for the Rp. 300 subsidy. Market shares are, respectively, 20.19, 20.52, and 21.06 %. Avoided electricity capacities in MW are 11.27 MW in the RP. 220 subsidy case, 12.82MW in the RP. 250 subsidy case and 15.37MW in the RP. 300 subsidy case.

All households, existing fans and new fans

In order to fully assess the energy saving potential of the SEEP, we need to add the estimated demand for new fans. In Tables 14 and 15, the combined total avoided electricity capacity for urban and rural households and for both existing and newly bought fans is shown.

Figure 1 pictures the projected total avoided electricity capacity for the same subsidies and for the Business-as-Usual (BAU) case. Results are in line with recent findings from the Climate Investment Fund[12], which raises many doubts concerning the effectiveness of the SEEP. Such effectiveness, however, does not just concern the amount of energy saved, but also the financial aspect of the scheme. The same little energy savings, as shown in Tables 14 and 15, and in Fig. 1, could have been obtained with a smaller investment. Our analysis shows that the funding needed, i.e., the fair value of the investment, would have been 40,350,101.59 or Rp. 2,649,435,089.70, approximately 19.3 % lower than what paid out by the Clean Technology Fund for subsidizing the production of energy efficient fans under the SEEP, i.e., around 50,000,000, or Rp. 3,283,060,000 (Climate Investment Funds 2013). In addition, since the actual subsidy is based on a bidding mechanism with multiple bidders[13], it is unlikely that the subsidy effectively paid out will be equal to 250 Rp., i.e., the highest possible. This will definitely reduce the costs of the scheme, together with its effectiveness, as our results show.

Conclusion

As can be seen by looking at Tables 14 and 15, while it is clear that the Super Efficient Equipment Program (SEEP) is capable of obtaining reductions in the consumption of energy, at the same time these are not of the amount one would expect and that has been foreseen by its promoters, i.e., 19,000 MW at the end of the third year, in particular in light of the fact that in the (near) future the number of electric fans per house is going to increase as population and per capita income increase. The reason for this is twofold. From one side, the maximum subsidy obtainable by the industry, and consequently the maximum discount applicable to the final price, is still low, i.e., Rp. 250 (3.81). Even when such maximum discount is applied, it is barely sufficient to cover, on average, the difference in price between energy efficient and

[11] 1 W = 0.000001 MW

[12] http://www.trust.org/item/20130802094324-4tug3/

[13] "The program is voluntary and manufacturers will bid for the amount of financial incentive as well as the total production quota through a reverse bidding mechanism with a pre-specified cap. The bidding mechanism is developed to allow multiple winners. The incentive will be paid per unit super-efficient fan to the manufacturer after the product leaves the factory for the market. A strict Monitoring and Verification (M&V) mechanism will check the quality and quantity of ceiling fans sold under the program" (Chunekar and Singh 2013).

regular, non-energy efficient, fans. Nevertheless, in such a market, a competitive price is extremely important. Since the discounted, after-subsidy price of energy efficient fans will still be slightly higher than the price of the non-energy efficient ones (on average, Rs. 1200 vs. Rs. 900, or \$18.28 vs. \$13.71), not many customers will orientate their expenditures towards the more efficient good. The second reason can be found in the price elasticity of demand for energy efficient fans, which proved not to be very high. In fact, demand can be considered fairly *inelastic*. This implies that reducing the price by 1 % does not cause an increase of 1 % in the quantity of energy efficient fans demanded. Recall in fact from Table 5 that $\beta_{R,1} = -0.8075$ and that $\beta_{U,1} = -0.6268$, which implies an increase (decrease) in quantity demanded of respectively 0.81 % for rural households and 0.63 % for urban households every 1 % decrease (increase) in the price of the good. What seems to be a necessary step from the Government is to intervene to change the price elasticity of demand for energy efficient goods. Hindering the production, and consumption, of inefficient fans can achieve this goal. An additional intervention could be the introduction and development of awareness and advertising campaigns, so to increase the knowledge about the differences between efficient and regular fans, in terms of costs and also in terms of the energy saving potential (Reddy and Shrestha 1998; Kumar and Sharma 1998; Liu et al. 2015). On the other hand, agencies such as the BEE and the Clean Technology Fund should develop econometric models in order to estimate the fair value of the scheme and afterwards to obtain more realistic projections in terms of its energy reduction potential. One example is the Super Efficient Equipment Program, developed by the Bureau of Energy Efficiency under the National Mission for Enhanced Energy Efficiency and partially funded by the Clean Technology Fund. As our empirical tests have shown, the subsidy may not be the most *effective* measure in obtaining the desired results. The sought-after diffusion of energy efficient fans cannot be obtained when the maximum discount offered does not reach the cost of the alternative, inefficient, product, in particular in a market where customers, specially in urban areas, seem not to have captured the distinction between efficient and regular items. What can be done here, is increasing the subsidy, since the amount of funding set aside by the Clean Technology Fund for the scheme is well above what is being currently spent with this level of subsidy, in the sense that the same results, in terms of diffusion of energy efficient fans, could have been obtained from an investment 19.3 % lower. Nevertheless, despite it limitations, the SEEP has to be intended as a market transformation program which has the goal to allow producers to switch in technology and to be able to market fans that they do not produce today.

Future research and improvements are needed, in particular for what concerns a more detailed specification of the equations used to estimate elasticities and a wider set of independent variables should be tested. Moreover, finding the *break-even* subsidy that allows the Super Efficient Equipment Program to reach the desired avoided electricity capacity of 19,000 MW proves to be a very important step. A strong labeling program, specifically designed for ceiling fans or that integrates the BEE's Standard and Labeling Program already in place, could be designed, as to help consumers make an informed choice about the potential savings obtainable from the purchase of a super efficient ceiling fan against a bigger sum to be spent. Additionally, what seems to be a key step is the introduction of minimum energy performance standards (MEPS), that could bring two desired results. First, removing from the market inefficient fans with high energy consumption; second, it could help concentrate the producers' financial efforts on more efficient fans, thus obtaining economies of scale and reducing their production cost. Finally, a subsidy could be designed around the real production cost of a super efficient ceiling fan, so to cover the fair price of their production and to incentivize producers to develop and market these appliances.

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Consent for publication No research activities involving Human Participants and/or other Animals have been or will be undertaken.

All the people and institutions of interest have been promptly informed.

A: Test statistics

A.1 Dickey-fuller unit-Root and normality tests for regressions

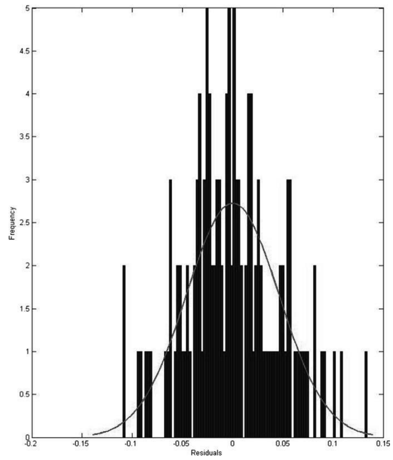
Table 16 Dickey-Fuller unit root tests

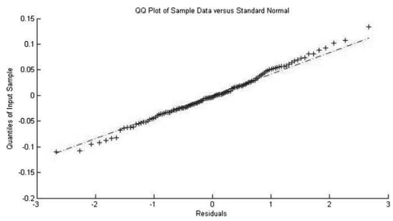
		Test statistic	Interpolated Dickey-Fuller		
			1 % Critical value	5 % Critical value	10 % Critical value
(a) $lnQ_{NE,t}$	$Z(t)$	−12.463	−3.500	−2.888	−2.578
	MacKinnon approximate p value for $Z(t) = 0.0000$				
(b)$lnQ_{EE,t}$	$Z(t)$	−12.463	−3.500	−2.888	−2.578
	MacKinnon approximate p value for $Z(t) = 0.0000$				
(c) lnQNE;R;t	$Z(t)$	−2.637	−3.497	−2.887	−2.577
	MacKinnon approximate p value for $Z(t) = 0.0855$				
(d) lnQEE;R;t	$Z(t)$	−5.228	−3.500	−2.888	−2.578
	MacKinnon approximate p value for $Z(t) = 0.0000$				
(e) lnQNE;U;t	$Z(t)$	−4.702	−3.497	−2.887	−2.577
	MacKinnon approximate p value for $Z(t) = 0.0855$				
(f) lnQEE;U;t	$Z(t)$	−13.879	−3.500	−2.888	−2.578
	MacKinnon approximate p value for $Z(t) = 0.0000$				
(g) lnPNE;t	$Z(t)$	−13.449	−3.500	−2.888	−2.578
	MacKinnon approximate p value for $Z(t) = 0.0000$				
(h) lnPEE;t	$Z(t)$	−8.532	−3.500	−2.888	−2.578
	MacKinnon approximate p value for $Z(t) = 0.0000$				
(i) lnMPCER;t	$Z(t)$	−4.141	−3.500	−2.888	−2.578
	MacKinnon approximate p value for $Z(t) = 0.0000$				
(j) lnMPCEU;t	$Z(t)$	−5.859	−3.500	−2.888	−2.578
	MacKinnon approximate p value for $Z(t) = 0.0000$				
(k) lnDIHH;t	$Z(t)$	−3.145	−3.497	−2.887	−2.577
	MacKinnon approximate p value for $Z(t) = 0.0234$				
(l) lnPopHH;t	$Z(t)$	−4.010	−3.500	−2.888	−2.578
	MacKinnon approximate p value for $Z(t) = 0.0000$				

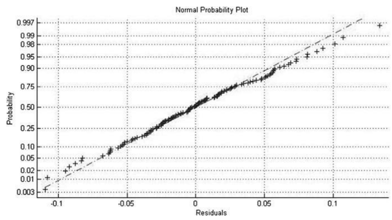
Table 17 Normality tests

	Test statistic	Critical value	p value	$H0$: Normality (0: cannot reject null)
(a) $[lnQ_{NE,t} = \alpha + \gamma_1 lnP_{NE,t} + \gamma_2 lnDI_{HH,t} + \gamma_3 lnPOP_{HH,t} + \epsilon_t]$				
Jarque - Bera	0.6237	5.5424	0.5000	0
Lilliefors	0.0510	0.0783	0.5000	0
(b) $[lnQ_{E,t} = \alpha + \beta_1 lnP_{E,t} + \beta_2 lnDI_{HH,t} + \beta_3 lnPOP_{HH,t} + \epsilon_t]$				
Jarque - Bera	2.2090	5.5424	0.2573	0
Lilliefors	0.0486	0.0783	0.5000	0

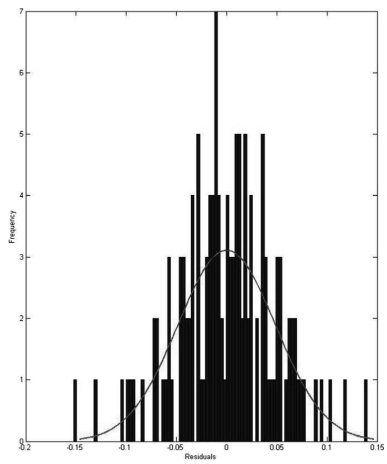
(a)

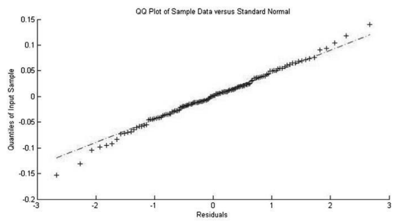


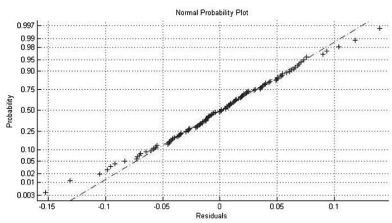




(b)







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Appendix E

Bruno Troja - Curriculum Vitae

Bruno Troja

Personal Details

Date of Birth	18.09.1980
Nationality	Italian

Work Experiences

March 2016 – Present	**II-VI Laser Enterprise, Zürich, Switzerland** *Business Analyst*
October 2014 – July 2015	**Ardian Private Equity (former AXA Private Equity), Zürich, Switzerland** *Mandates Solutions Manager*
September 2010 – February 2016	**University of Zurich, Department of Banking and Finance** *Research and Teaching Assistant*
January 2013 - April 2013	**Institute for Global Environmental Strategies (IGES), Hayama, Japan** *Research Assistant, Intern*
August 2004 – August 2014	**Azienda Agricola "Bruno Troja", Avola, Italy** *Owner/Entrepreneur*

Education

September 2010 – July 2016	**PhD Studies in *Finance*, University of Zurich** Valuation of Investments under Uncertainty, Quantitative Finance and Risk Management, Econometric Analysis, Project Management, Business Analysis, Sustainable Investments.
April 2015 – Present	**Sommelier Qualification (Association Suisse des Sommeliers Professionnels, ASSP)**
2010	**Master of Advanced Studies in Quantitative Finance, University of Zurich and ETH Zurich** *Magna Cum Laude (5.26/6)* Focus: Finance, Quantitative Finance, Risk Management, Sustainable Finance.
2007	**Bachelor & Master in Business Administration and Political Economics, University of Catania, Catania, Italy** *Summa Cum Laude(108/110)* Focus: Economics, Business Administration, Management.